Slow Debt, Deep Recessions*

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Abstract

Business credit lags GDP growth by about one year. This contributes to high leverage during recessions and slow deleveraging. We show that a model in which firms use risky long-term debt replicates this slow adjustment of firm debt. In the model, slow-moving debt has important effects for real activity. High levels of firm debt issued during expansions are only gradually reduced during recessions. This generates an adverse feedback loop between high default rates and low investment and thereby amplifies the downturn. Sluggish deleveraging slows down the recovery. The equilibrium is constrained inefficient because firms exert an externality on the holders of previously issued debt. The constrained efficient allocation substantially reduces macroeconomic volatility.

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1. Introduction

Firm debt follows GDP with a lag. Peaks and troughs in firm credit growth tend to occur about one year later than in GDP. While GDP growth turned negative in 2008, US firm debt continued to grow at a robust annual rate of more than five percent. This slow adjustment of debt is potentially important because it contributes to high levels of debt during recessions and a prolonged deleveraging process. In this paper, we show that slow-moving firm debt can generate deep recessions and slow recoveries.

Using firm-level data, we find that the slow adjustment of debt is related to the extent to which firms use long-term debt. We then build a business cycle model of production, firm financing, and costly default in which firms can borrow long-term. The model replicates the empirical co-movement between firm debt and output. Risky long-term debt is crucial for this result. When firms borrow long-term, high debt levels during a boom carry over into the subsequent recession. During the downturn, firms are reluctant to reduce the high debt levels inherited from the past because the benefits of this reduction would mostly fall to creditors. As a result, firm leverage, default rates, and credit spreads all peak during recessions, in line with the empirical evidence.

The rise in credit spreads during downturns drives up firms' cost of capital and induces them to cut back on investment. Firm assets fall at a faster rate than debt because firms adjust debt slowly. The resulting rise in leverage causes an additional increase in default risk and credit spreads which depresses investment even further. In this way, slow-moving debt gives rise to an adverse feedback loop between high default rates and low investment which amplifies and prolongs the downturn.

Importantly, the additional volatility in output generated by slow-moving debt is inefficient. Firms adjust debt slowly because they do not internalize all associated costs. They exert an externality on the holders of previously issued debt who bear a large part of the costs of elevated credit risk. In the constrained efficient allocation, debt is adjusted immediately in response to aggregate shocks. This avoids the strong increase in default rates and credit spreads during downturns and thereby substantially reduces macroeconomic volatility.

We begin our analysis in a simple two-period version of our model. This allows us to characterize the key mechanisms analytically. We then proceed to a quantitative analysis of a dynamic business cycle model in which firms choose capital, labor, leverage, and debt maturity. We show that the model successfully replicates the empirical lag structure between firm debt and output. The model is also successful in generating counter-cyclical leverage, default risk, and credit spreads, as well as a pro-cyclical term structure in credit spreads. We compare the decentralized equilibrium to the constrained efficient allocation and find that slow debt substantially increases output volatility. This implies room for welfare improving stabilization policies.

Our paper contributes to a large literature which studies the role of firm debt for cyclical fluctuations. The standard approach in this literature is to model all firm debt as short-term, i.e. all debt issued in period t fully matures in period $t+1$. From an empirical point of view, the disregard of long-term debt is problematic. About 75% of US corporate debt does not mature within the next year. At issuance, the average term to maturity is three to four years for bank loans, and more than eight years for corporate bonds (Adrian, Colla, and Shin, 2013). Our results suggest that models which take firms' use of long-term debt into account can contribute to our understanding of cyclical fluctuations and effective stabilization policies.

Computational difficulties are the main reason why risky long-term debt is usually absent from dynamic macroeconomic models. Optimal firm behavior depends on the price of long-term debt, which itself depends on firm behavior both today and in the future. A firm that cannot commit to future actions must take into account how today's choices will affect future firm behavior. In this paper, we compute the global solution to this fixed point problem. This allows us to study how firms optimally adjust their debt structure over time and how these choices shape the business cycle.

The paper most closely related to ours is Gomes, Jermann, and Schmid (2016) who use first-order perturbation methods to study a New Keynesian business cycle model with risky long-term debt in which firms' borrowing and investment decisions exert an externality on existing creditors. Their main result is that shocks to inflation change the real burden of outstanding nominal long-term debt and thereby generate persistent cyclical variations in leverage and investment. We show that a model with risky long-term debt is successful in replicating the empirical lag structure between firm debt and output, and that slow debt gives rise to amplified and prolonged macroeconomic fluctuations which are constrained-inefficient. We derive these results by studying a fully non-linear global solution of a flexible-price model with productivity shocks. A new feature is that firms simultaneously issue short- and long-term debt. Endogenous debt maturity allows firms to respond to the distortions introduced by risky long-term debt.[1]

Our paper also relates to Cooley, Marimon, and Quadrini (2004) who study a model of long-term financing with limited commitment. One key difference is that in their model the commitment problem becomes more severe during booms while default does not occur in equilibrium. In our model, the commitment problem endogenously becomes more severe during downturns which renders the default rate and credit spreads countercyclical and gives rise to amplification.[2]

More generally, our paper relates to the broader literature on the role of firm debt for cyclical fluctuations. In this literature, all firm debt is short-term (e.g. Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999; Jermann and Quadrini, 2012; Khan and Thomas, 2013; Christiano, Motto, and Rostagno, 2014; Gilchrist, Sim, and Zakrajšek, 2014; Arellano, Bai, and Kehoe, 2019). In these short-term debt models, high debt levels during a boom do not carry over into the subsequent recession and there is no link between slow debt and deep recessions.

[1] Also related is Miao and Wang (2010) who study a model of long-term debt in which firms do not anticipate that current debt issuance affects future firm behavior. Poeschl (2018) focuses on firms' optimal maturity choice. Crouzet (2017), Karabarbounis and Macnamara (2019), and Jungherr and Schott (2020) study long-term debt in models without aggregate shocks.

[2] Occhino and Pescatori (2015) analyze the effect of an exogenous stock of existing short-term debt on firm behavior. Caggese and Perez (2016) and Paul (2018) study amplification in environments where long-term debt levels are fixed.

Another feature of standard financial accelerator models is that financing matters because firm net worth is scarce and equity issuance is costly (or ruled out completely). In contrast, there are no social costs of equity issuance in our model. Net worth falls sharply during downturns because firms are unwilling to raise equity from shareholders if the associated benefits would mainly go to creditors. This mechanism causes leverage, default rates, and credit spreads to rise during downturns and thereby amplifies the recession.[3]

The commitment problem generated by risky long-term debt is the focus of several contributions in the literature on sovereign default (e.g. Arellano and Ramanarayanan, 2012; Chatterjee and Eyigungor, 2012; Hatchondo, Martinez, and Sosa-Padilla, 2016) and corporate finance (e.g. Admati, DeMarzo, Hellwig, and Pfleiderer, 2018). Aguiar, Amador, Hopenhayn, and Werning (2019) and DeMarzo and He (2020) show that risky long-term debt induces sovereigns and firms to adjust debt slowly over time. The key distinctive feature of our analysis relative to this work is endogenous output and investment. Because slow debt drives up firms' cost of capital during downturns, it generates deep recessions and slow recoveries.[4]

In Section 2, we establish empirical facts about the dynamic co-movement between firm debt and output. Section 3 provides analytical results on slow debt and deep recessions in a two-period setup. Section 4 presents the main results of the paper. We study a quantitative business cycle model of production, firm financing, and costly default, and we compare the decentralized allocation to its constrained efficient counterpart. Concluding remarks follow.

2. Empirical Facts

In this section, we document several empirical facts about the relationship between US firm credit and the business cycle. In particular, we show that firm credit lags output by about one year and that this lag is related to the extent to which firms use long-term debt.

The upper panel of Figure 1 shows aggregate leverage of US non-financial firms together with the growth rate of real GDP. Leverage peaks during recessions and deleveraging takes a considerable amount of time. After the 2008-09 recession, GDP growth had returned to its 2007 level by 2010, while leverage had not yet returned to its 2007 level by 2015. One reason why leverage peaks during recessions is displayed in the lower panel of Figure 1: Firm debt follows GDP with a lag. Peaks and troughs in firm credit growth tend to occur about one year later than in output. When GDP

[3] In financial accelerator models with short-term debt, first moment shocks often generate a procyclical default rate which is at odds with the data (see e.g. Carlstrom and Fuerst, 1997; Gomes, Yaron, and Zhang, 2003; Covas and Den Haan, 2012).

[4] In practice, market participants try to mitigate the commitment problem generated by risky long-term debt through various contracting features such as seniority structures (as in Chatterjee and Eyigungor, 2015) or debt covenants (e.g. Xiang, 2019). See Appendix D for a brief discussion of the empirical literature on seniority and debt covenants.

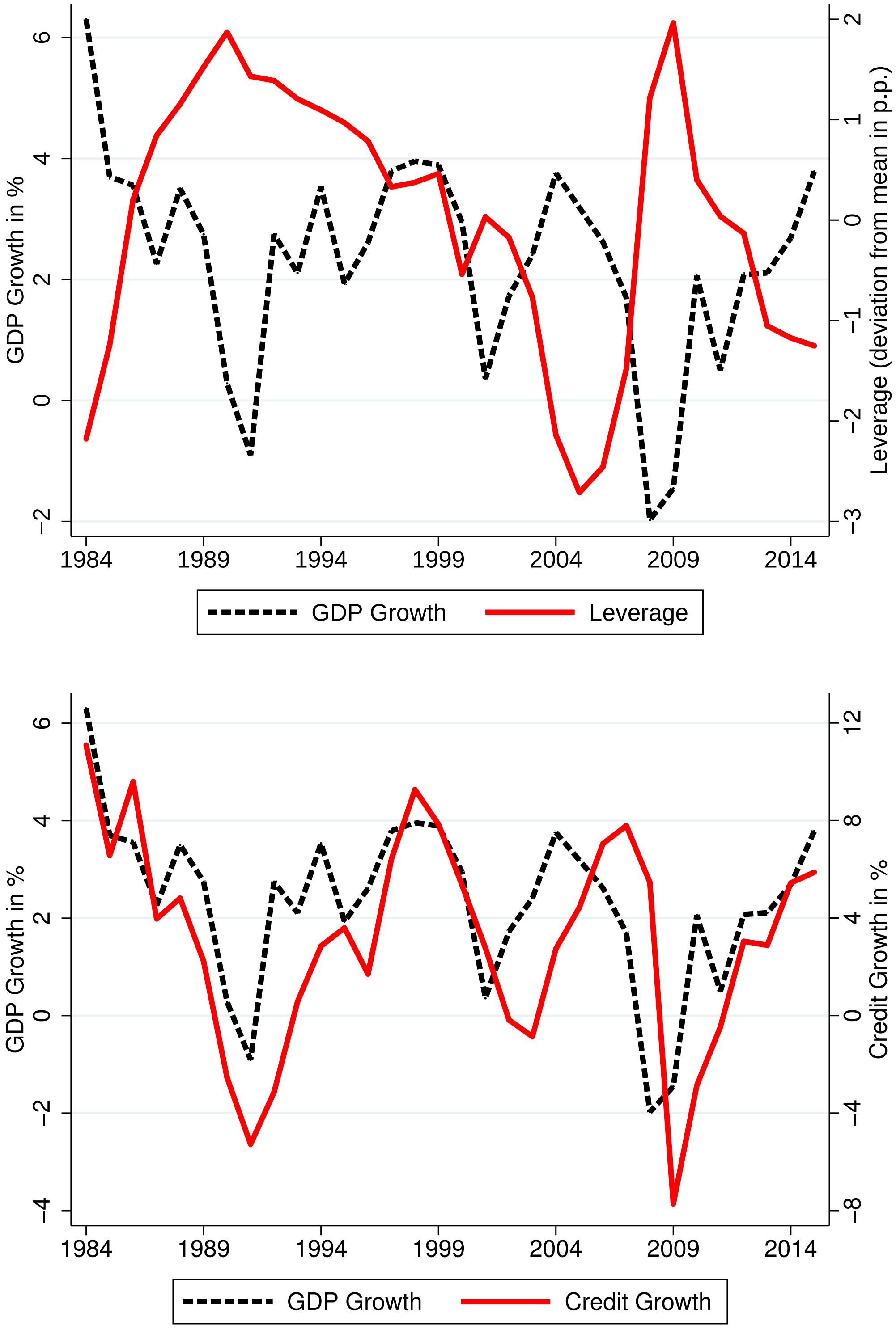


Figure 1: Leverage and Credit Growth

Note: Leverage (upper panel, solid red line, right axis) is total debt of non-financial firms divided by total assets (book value, marked-to-market). *GDP Growth* (dashed black line, left axis) is annual growth of real GDP. *Credit Growth* (lower panel, solid red line, right axis) is annual growth of real total debt of non-financial firms. Stock variables are end-of-year. Data source: Flow of Funds. See Appendix A for details.

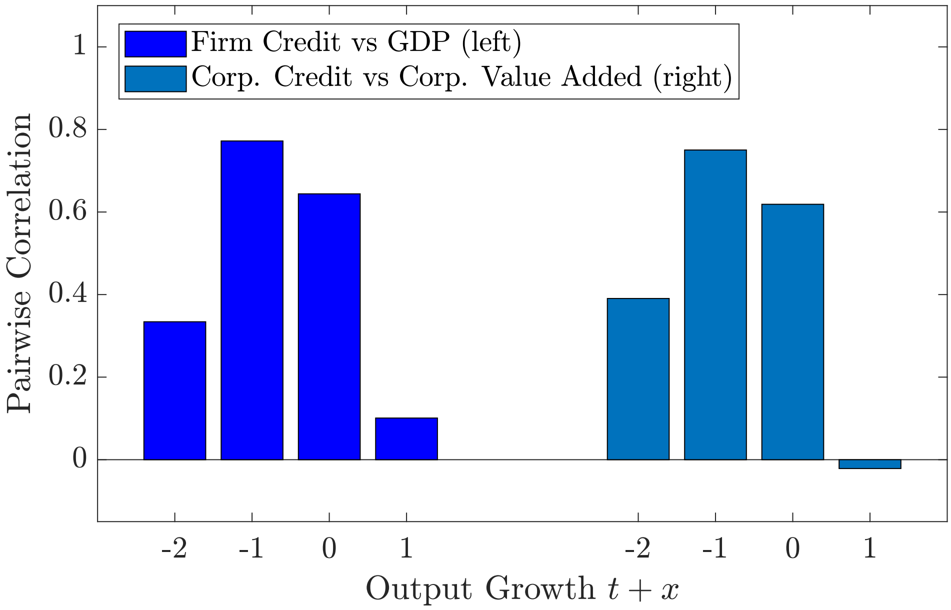


Figure 2: Correlations Firm Credit Growth t with Output Growth $t+x$ (Flow of Funds)

Note: Bars show pairwise correlations. The left bars show correlations between annual growth of real total debt of non-financial firms at the end of year t and real GDP growth in year $t+x$. The right bars show correlations between annual growth of real total debt of non-financial corporate firms at the end of year t and real growth of non-financial corporate value added in year $t+x$. Data comes from the Flow of Funds 1984-2015. See Appendix A for details.

growth turned negative in 2008, firm credit continued to grow at an annual rate of more than five percent.

This dynamic co-movement between output and firm credit is examined more formally in Figure 2. The left side shows pairwise correlations between total firm credit growth in year t and GDP growth in year $t+x$. Because we use data from the corporate sector to inform our quantitative model, we also show correlations between corporate credit and corporate value added in the right half of Figure 2. In both cases, the correlations highlight the slow-moving behavior of debt. While the contemporaneous correlation between growth in firm credit and output is positive, firm credit is most strongly correlated with output growth one year ago.[5][6]

The slow adjustment of debt is potentially important because it contributes to high levels of debt during recessions and a prolonged deleveraging process which could slow down the recovery. Using firm-level data, we show that this lag in credit is related to firms' use of long-term debt. From Compustat, we extract balance sheet information on

[5]Using quarterly data, we find that the correlation between firm credit and GDP growth peaks at a lag of five to six quarters (seven quarters for the correlation between corporate credit and corporate value added). See Appendix A.

[6]The behavior of firm leverage is determined by changes in the numerator (debt) and the denominator (assets). We also calculate the dynamic co-movement between output and firm assets. In contrast to debt, no lag is present for firm assets. See Appendix A for details.

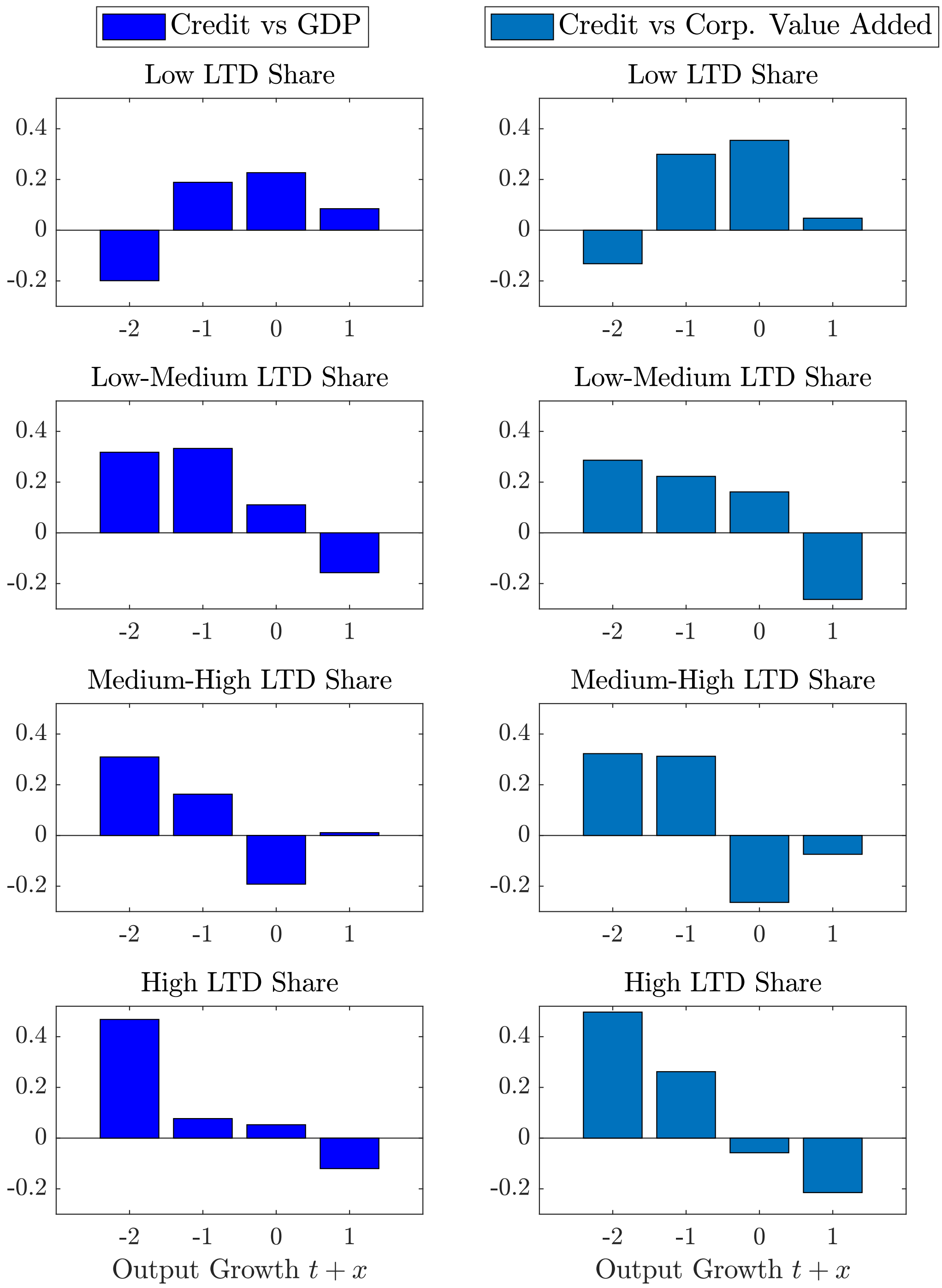


Figure 3: Correlations Firm Credit Growth t with Output Growth $t + x$ (Compustat)

Note: Bars show pairwise correlations between annual growth of real total credit of non-financial Compustat firms at the end of year t and real GDP growth in year $t + x$ (*left*), and real growth of corporate value added in year $t + x$ (*right*). Credit is total debt of all firms in the respective quartile of the long-term debt (LTD) share distribution. The LTD share is debt due in more than one year divided by total debt. Time period: 1984-2015. Data is from Compustat (firm debt, LTD share) and from the Flow of Funds. See Appendix A for details.

a large panel of publicly traded non-financial firms. We repeat the above exercise for firms with different shares of long-term debt, i.e. the share of debt with remaining term to maturity of more than one year. In every year, we sort firms into quartiles based on their share of long-term debt. For each group, we calculate total firm debt and show the correlations with output growth in year $t + x$.

The results are shown in Figure 3. Output is measured as GDP (left panels) and corporate value added (right panels). The panels in the top row of Figure 3 show correlations for the quartile of firms with the lowest long-term debt shares. The rows below show the corresponding correlations for firms with higher shares of long-term debt.

The figure shows that the lag of firm credit with respect to output is more pronounced for firms with a higher long-term debt share. For firms in the lowest quartile, credit co-moves most strongly with contemporaneous output growth. For firms with longer debt maturities, firm credit co-moves more and more strongly with lagged output. For firms in the second quartile of the long-term debt share distribution, the correlation peaks at the first lag of GDP growth. For firms in the third and fourth quartile, the co-movement is strongest with output growth two years ago.[7]

This firm-level evidence suggests that the slow response of firm credit to changes in output is related to firm's use of long-term debt. In order to study the role of slow debt for the business cycle, a model of long-term debt is needed.

3. Two-period Model

We begin our analysis in a simple two-period environment. This allows us to describe analytically how an existing stock of long-term debt can give rise to slow debt and deep recessions.

A risk-neutral firm produces output using capital and labor. Investment is financed with equity and debt. The optimal capital structure solves a trade-off between the tax advantage of debt and the expected cost of default. The firm decides on its scale of production and the preferred capital structure in the presence of previously issued long-term debt. This variable is exogenous in the two-period setup. It will be endogenized in the dynamic business cycle model of Section 4.

3.1. Setup

There are two periods: $t = 1, 2$. Consider a firm owned by risk neutral shareholders. In period 2 the firm uses capital k and labor l to produce output y according to:

$$y = z\left(k^{\psi} l^{1-\psi}\right)^{\zeta}, \quad \text{with} \quad \zeta, \psi \in (0, 1). \tag{1}$$

[7] Alternative firm-level criteria used to group firms into different sub-samples (e.g. total assets, total debt, leverage, profitability, or asset liquidity) do not generate systematic patterns with respect to the lag in firm debt relative to output.

The firm chooses capital and labor in period 1. Productivity z is known at this point. Firm earnings are uncertain because of a capital quality shock ε. This is the only source of uncertainty. Earnings before interest and taxes in $t = 2$ are given by

$$y + \varepsilon k - wl - \delta k, \tag{2}$$

where w is the wage rate and δ is depreciation. At $t = 1$, ε is a random variable with probability density $\varphi(\varepsilon)$. An example for a negative capital quality shock is an unforeseen change in technology or consumer demand which reduces the value of existing firm-specific capital.

There are two ways of financing capital: equity and debt.

Definition: Debt. A debt security is a promise to pay one unit of the numéraire good together with a fixed coupon payment c at the end of period 2.

Let p be the market price of a one-period bond sold by the firm in period 1. If the firm sells a number Δ of new bonds, it raises an amount $p\Delta$ on the bond market. This newly issued debt matures in period 2. In addition, an exogenous number b of bonds is due in period 2. One may think of b as long-term debt which has been issued before period 1. The stock of total debt in period 2 becomes $\tilde{b} = b + \Delta$.

An alternative to debt financing is equity issuance, denoted as e. This is the net cash flow from shareholders to the firm. With a stock of capital q in place at the beginning of period 1, the capital stock k in period 2 becomes

$$k = q + e + p\Delta = q + e + p(\tilde{b} - b). \tag{3}$$

Firm earnings are taxed at rate τ. Debt coupon payments are tax deductible. The firm's stock of equity after production and repayment of debt in period 2 is

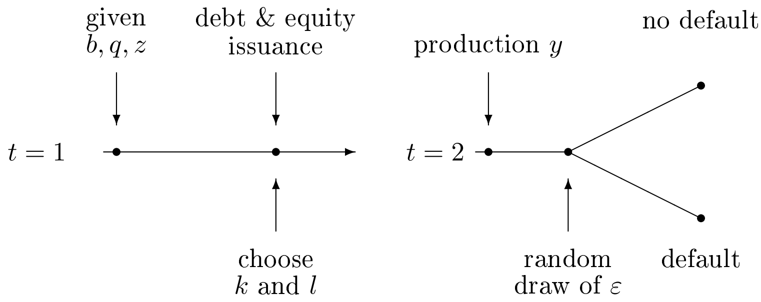
$$\tilde{q} = k - \tilde{b} + (1 - \tau)(y + \varepsilon k - wl - \delta k - c\tilde{b}). \tag{4}$$

The fact that coupon payments are tax deductible lowers the total tax payment by the amount $\tau c\tilde{b}$. This is the benefit of debt. The downside is that the firm cannot commit to repaying its debt after firm earnings are realized in period 2.

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. Default is costly. A fixed fraction ξ of firm assets is lost in this case.

The timing is summarized in Figure 4. In period 1, the firm starts with given levels of debt b, capital q, and productivity z. The firm chooses capital k and labor l. Capital is financed through equity issuance e and the revenue from selling additional bonds $p(\tilde{b}-b)$. In period 2, production takes place. After the realization of the capital quality shock ε, the firm decides whether to default.

Figure 4: Two-period Model - Timing



3.2. Firm Problem

The firm maximizes shareholder value. Because shareholders are risk neutral, the firm's objective is the expected present value of net cash flows to shareholders.

We solve the firm's problem using backward induction, beginning with the default decision after the realization of firm earnings in period 2. Limited liability protects shareholders from large negative realizations of ε. There is a unique threshold realization $\bar{\varepsilon}$ which sets equity after production equal to zero:

$$\bar{\varepsilon}: \quad \tilde{q} = 0 \quad \Leftrightarrow \quad k - \tilde{b} + (1-\tau)(y + \bar{\varepsilon}k - wl - \delta k - c\tilde{b}) = 0 \tag{5}$$

If ε is smaller than $\bar{\varepsilon}$, full repayment would result in negative equity whereas default provides an outside option of zero. In this case, the firm optimally defaults on its liabilities. The threshold value $\bar{\varepsilon}$ is increasing in total debt $\tilde{b}$ and falling in capital k. By choosing the ratio of debt $\tilde{b}$ to capital k in period 1, the firm controls the default threshold $\bar{\varepsilon}$ and thereby the probability of default.

In period 1, the firm decides on its scale of production and its preferred financing mix between equity and debt. The firm anticipates that shareholders receive $\tilde{q}$ whenever $\varepsilon \geq \bar{\varepsilon}$ and zero otherwise:

$$\max_{k,l,e,\tilde{b},\bar{\varepsilon}} \quad -e + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty} \left[k - \tilde{b} + (1-\tau)(y + \varepsilon k - wl - \delta k - c\tilde{b})\right] \varphi(\varepsilon) d\varepsilon \tag{6}$$

$$\text{subject to:} \quad y = z\left(k^{\psi} l^{1-\psi}\right)^{\zeta}$$

$$\bar{\varepsilon}: \quad 0 = k - \tilde{b} + (1-\tau)(y + \bar{\varepsilon}k - wl - \delta k - c\tilde{b})$$

$$k = q + e + p(\tilde{b} - b),$$

where r is the risk-free interest rate. The optimal firm policy crucially depends on the bond price p. A high bond price implies a low credit spread which reduces the firm's cost of capital. We derive the firm-specific bond price from the creditors' optimization problem.

3.3. Creditors' Problem

Creditors are risk neutral and discount the future at the same rate $1/(1+r)$ as shareholders. They buy firm bonds in period 1. If the firm does not default in period 2, creditors receive full repayment. In case of default, they receive the firm's liquidation value $(1-\xi)\underline{q}$, where

$$\underline{q} \equiv k + (1-\tau)(y + \varepsilon k - wl - \delta k). \tag{7}$$

Creditors are perfectly competitive and break even on expectation. The break-even price of firm debt depends on the probability $\Phi(\bar{\varepsilon})$ that the firm defaults in period 2:

$$p = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\;\varphi(\varepsilon)d\varepsilon\right] \tag{8}$$

The credit spread is defined as the excess return on firm debt (conditional on full repayment) over the riskless rate: $(1+c)/p-(1+r)$. If creditors expect a positive probability of default, they will charge a positive spread.

3.4. Equilibrium

We solve for the partial equilibrium allocation for a given wage w and a given risk-free rate r. The firm maximizes shareholder value (6) subject to creditors' break-even condition (8). As we show in Appendix B, this problem can be simplified by re-writing it in terms of only two endogenous variables: the scale of production determined by firm capital k and the default threshold $\bar{\varepsilon}$. Accordingly, an interior solution is characterized by two first order conditions.

3.4.1. First Order Conditions

For analytical tractability, in this part we consider the special case of $\xi = 1$. This means that the liquidation value of the firm is zero in case of default and the bond price in (8) only depends on $\bar{\varepsilon}$:

$$p = \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]. \tag{9}$$

The firm's first order condition with respect to capital k is:

$$\underbrace{-1}_{\substack{\text{Marginal}\\ \text{cost of capital}}} + \underbrace{\frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1+(1-\tau)\left(\text{MPK}-\delta+\bar{\varepsilon}\right)}{1+(1-\tau)c}}_{\substack{\text{Marginal increase in market value}\\ \text{of newly issued debt } p(\tilde{b}-b)}} + \underbrace{\frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}(\varepsilon-\bar{\varepsilon})\varphi(\varepsilon)d\varepsilon}_{\substack{\text{Marginal increase in}\\ \text{expected stock of equity } \tilde{q}}} = 0 \tag{10}$$

A marginal increase in k has an opportunity cost of one. The marginal benefit consists of an increase in the market value of newly issued debt $p(\tilde{b}-b)$ and in the expected stock of firm equity after production $\tilde{q}$. Shareholders benefit from a high market value of newly issued debt because less equity issuance e is required to finance a given level of capital k. The marginal product of capital MPK is increasing in z and falling in k.

The first order condition for an optimal choice of the default probability $\bar{\varepsilon}$ is:

$$\underbrace{[1-\Phi(\bar{\varepsilon})]\frac{\partial\tilde{b}}{\partial\bar{\varepsilon}}\tau c}_{\text{Marginal tax benefit of } \bar{\varepsilon}} \quad - \quad \underbrace{\varphi(\bar{\varepsilon})(1+c)(\tilde{b}-b)}_{\substack{\text{Marginal increase in} \\ \text{expected cost of default} \\ \text{internalized by the firm}}} = 0 \tag{11}$$

The first term is the marginal benefit of an increase in $\bar{\varepsilon}$. A higher value of $\bar{\varepsilon}$ implies a higher debt level $\tilde{b}$. If default is avoided, this is beneficial as it costs shareholders only $(1-\tau)c$ to increase the promised payment to creditors by c. Because competitive creditors break even, the entire tax benefit of debt is captured by shareholders.

The second term in (11) plays a key role in this model. It shows the firm's marginal cost of an increase in $\bar{\varepsilon}$. A higher value of $\bar{\varepsilon}$ increases the default probability by $\varphi(\bar{\varepsilon})$ which lowers the bond price and therefore the market value of newly issued debt. In case of default, creditors lose the entire amount $(1+c)\tilde{b}$ (because $\xi = 1$). Through the bond market, the firm internalizes that the buyers of newly issued debt lose $(1+c)(\tilde{b}-b)$ in case of default. However, the firm disregards all potential losses which accrue to the holders of previously issued debt. There is a gap between the firm's marginal cost of $\bar{\varepsilon}$ and the social cost. As we show below, this has important implications for firm behavior and the business cycle.

3.4.2. Slow Debt and Deep Recessions

We characterize the equilibrium of this economy using comparative statics. All proofs are deferred to Appendix B. The first proposition describes the effect of existing long-term debt on the firm's choice of debt $\tilde{b}$, holding the level of capital fixed.

Proposition 1. ***Slow debt:*** *For a given level of capital k, the firm's choice of debt $\tilde{b}$ is increasing in the stock of existing debt b.*

Consider the firm's decision to issue one additional bond in period 1. This would increase the probability of default in period 2 and lower the expected payoff for all creditors. However, through the bond market the firm only internalizes the expected losses which accrue to the buyers of newly issued debt. With a higher stock of existing debt b, a larger part of the increase in expected default costs is shared with existing creditors. This allows the firm to enjoy a given amount of the tax benefits of debt at a lower private cost. As a result, the firm optimally decides to utilize the tax benefit of debt more intensively by choosing higher values of $\bar{\varepsilon}$ and $\tilde{b}$.

Proposition 1 implies that the firm's choice of debt $\tilde{b}$ is history-dependent. For any given amount of capital, the firm chooses a higher debt level $\tilde{b}$ if the stock of existing debt is larger. Note that the firm's first order condition (11) can never hold if $\tilde{b} < b$ (as the left hand side of (11) is strictly positive in this case). This means that debt repurchases (i.e. choosing $\tilde{b} < b$) are never optimal for the firm. The benefits from active deleveraging would entirely fall to the holders of existing debt in the form of lower default risk and a higher bond price. At the same time, the firm would drive up the price of the outstanding debt which it repurchases and shareholders would lose part of the tax benefit of debt (Admati et al., 2018).

For the following propositions, we allow capital to adjust optimally according to the first order condition in (10).

Proposition 2. ***Cyclicality of leverage, default risk, and credit spread:*** *If $b = 0$, leverage, default risk, and the credit spread are constant in productivity z. If $b > 0$ and capital increases in z, default risk and the credit spread are decreasing in z. Leverage is decreasing in z if ζ is sufficiently close to one.*

A higher level of productivity z increases output. Proposition 2 states that without existing long-term debt, leverage, default risk, and credit spreads are acyclical. The reason is that changes in capital k are proportional to changes in debt $\tilde{b}$ if $b = 0$. Productivity z affects the firm's optimal scale of production but not leverage or the risk of default.[8]

This result changes for $b > 0$. Dividing the firm's first order condition (11) by k shows that, compared to the case when $b = 0$, the marginal cost of an increase in $\bar{\varepsilon}$ is reduced by the term

$$\varphi(\bar{\varepsilon})(1 + c)\frac{b}{k}\,.$$

It denotes the part of the marginal increase in the expected cost of default which is disregarded by the firm. This term is scaled by the ratio b/k. A large stock of existing debt b relative to capital k provides a strong incentive to increase the risk of default.

Changes in productivity z affect the choice of k, whereas existing debt b is fixed. As a result, the ratio b/k increases when k falls. This implies that a given stock of existing debt increases the default risk by more during downturns than during expansions. The firm's default risk and credit spread become counter-cyclical. If the curvature parameter ζ is sufficiently close to one, leverage is counter-cyclical as well.[9]

Proposition 2 demonstrates that a model with risky long-term debt can generate the counter-cyclical behavior of default risk, credit spreads, and leverage observed in the data. A counter-cyclical default rate implies that the firm's cost of capital increases during downturns. Proposition 3 states that this mechanism can amplify output fluctuations.

[8] The optimal scale of production depends on the *marginal* product of capital, while optimal leverage depends on expected default costs which increase with the *average* product of capital. Given the assumed production technology, the marginal product is always proportional to the average product. As shown in Appendix B, this implies that optimal leverage does not vary with z of k in the benchmark case of $b = 0$.

[9] Ceteris paribus, higher default risk implies higher leverage but this effect is counteracted if the fall in z reduces the firm's average product of capital. As we show in Appendix B, increased default risk discourages investment and thereby imposes a lower bound on the firm's marginal product of capital. Because the marginal product is approximately equal to the average product for $\zeta \to 1$, this implies that leverage increases together with default risk during a downturn.

Proposition 3. ***Deep recessions:*** *If $b > 0$ and ζ is sufficiently close to one, then a fall in capital k and output y which is caused by a drop in productivity z is amplified by the resulting increase in default risk.*

If $b > 0$, default risk increases when capital falls, which drives up expected default costs and the credit spread. This increases the firm's cost of capital and can thereby amplify the fall in investment and production.[10]

Taken together, Propositions 2 and 3 describe an adverse feedback loop between high default rates and low investment. If a firm chooses a low amount of capital, a positive stock of previously issued debt implies that the default rate and the credit spread increase (Proposition 2). The higher cost of capital can further depress investment (Proposition 3) which in turn drives up the default rate even more, and so on.

The described mechanism is very general. While we consider changes in (revenue) productivity, any initial change in k induced by various kinds of demand and supply shocks can be amplified in the way described above.

3.5. Constrained Efficiency

The output response to changes in z is amplified because of the firm's disregard for the effects of its actions on the value of existing debt. A social planner who values the payoffs to all agents (shareholders *and* creditors) would maximize firm value (i.e. the value of all equity and debt claims) instead of shareholder value as in (6). Accordingly, the planner's objective reads as:

$$pb - e + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty}\Big[k - \tilde{b} + (1-\tau)(y + \bar{\varepsilon}k - wl - \delta k - c\tilde{b})\Big]\varphi(\varepsilon)d\varepsilon \tag{12}$$

The constrained efficient allocation maximizes (12) subject to the same constraints as in the firm problem (6) and creditors' break-even condition (8). Proposition 4 states that both the history dependence of debt and the amplified output response due to counter-cyclical default are absent in this case.

Proposition 4. ***Constrained efficiency:*** *If the firm internalizes the value of existing debt as in* (12)*, the choice of debt $\tilde{b}$ is independent of b. Leverage, default risk, and the credit spread are constant in productivity z for any value of b.*

The allocation described by Proposition 4 is markedly different from the equilibrium described above. Neither slow debt nor deep recessions arise. The difference between the equilibrium described above and the constrained efficient allocation is entirely due to the stock of existing debt b. It is therefore important to endogenize firms' choice of long-term debt in a dynamic model. This is the model we study in Section 4.

[10] If $b = 0$, leverage and default risk are chosen to maximize the *average* return on capital net of taxes and default costs. Because of decreasing returns to scale, the *marginal* return net of taxes and default costs is maximized at a higher value of leverage and default risk. This implies that for low values of ζ, a small increase in leverage and default risk can locally encourage investment (as shown in Appendix B). For $\zeta \to 1$, the gap between the firm's average and its marginal return disappears and the increase in default risk unambiguously amplifies the fall in k and y.

4. Business Cycle Model

After having established analytical results on slow debt and deep recessions, we now proceed to a dynamic open economy business cycle model of production, leverage, and debt maturity. The main additional feature is that firms can now sell short- and long-term bonds. The stock of existing debt b becomes endogenous: The amount of long-term debt issued today determines the stock of existing debt next period.

We introduce a linear issuance cost for new bonds. Short-term debt needs to be constantly rolled over, which implies high issuance costs. Long-term debt allows maintaining a given stock of debt at a lower level of bond issuance. The downside of long-term debt is that it increases the future amount of outstanding debt which raises future default risk.

4.1. Firm Setup

A firm i uses capital k_{it} and labor l_{it} to produce output according to:

$$y_{it} = z_t \left(k_{it}^{\psi} l_{it}^{1-\psi} \right)^{\zeta}, \quad \text{with:} \quad \zeta, \psi \in (0,1) \tag{13}$$

The natural logarithm of aggregate (revenue) productivity z_t follows an AR(1) process and is realized at the end of period $t-1$. Earnings before interest and taxes are given as

$$y_{it} + \varepsilon_{it} k_{it} - w_t l_{it} - \delta k_{it} - f, \tag{14}$$

where f is a fixed cost of operation. The firm-specific idiosyncratic capital quality shock ε_{it} is i.i.d. and follows a continuous probability distribution $\varphi(\varepsilon)$.

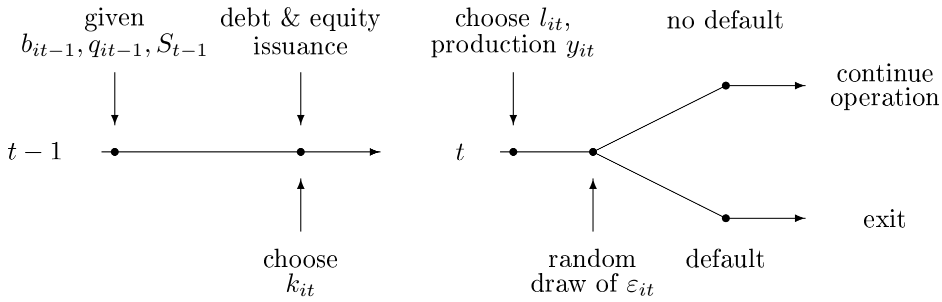
The firm can finance capital with equity, short-term debt, and long-term debt.

Definition: Short-term Debt. A short-term bond issued at the end of period $t-1$ is a promise to pay one unit of the numéraire good in period t together with a fixed coupon payment c. The number of short-term bonds sold by firm i and due in period t is $\tilde{b}_{it}^{S}$.

Definition: Long-term Debt. A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon payment c in period t. In addition, the firm repays a fraction $\gamma \in (0,1)$ of the principal in period t. In period $t+1$, a fraction $1-\gamma$ of the bond remains outstanding. The firm pays a coupon payment $(1-\gamma)c$ and repays the fraction γ of the remaining principal. In this manner, payments decay geometrically over time. The maturity parameter γ controls the speed of decay. The number of long-term bonds chosen by the firm at the end of period $t-1$ is $\tilde{b}_{it}^{L}$.

This computationally tractable specification of long-term debt goes back to Leland (1994). Short-term debt and long-term debt are of equal seniority.

Figure 5: Business Cycle Model - Timing



Definition: Issuance cost. The firm pays an amount η for each new short-term or long-term bond sold. Repurchasing outstanding long-term debt (by choosing $\tilde{b}^L_{it} < b_{it-1}$) is costless. The total issuance cost $H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1})$ is therefore

$$H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1}) = \eta\Big(\tilde{b}^S_{it} + \max\{\tilde{b}^L_{it} - b_{it-1}, 0\}\Big), \tag{15}$$

where b_{it-1} is the stock of previously issued long-term bonds outstanding before the firm decides on its investment and financing policy at the end of period $t-1$.

This issuance cost makes short-term debt unattractive because it needs to be constantly rolled over which implies high issuance costs.

The firm finances its capital stock by injecting equity and by selling new short- and long-term bonds. Let q_{it-1} be the stock of assets in place before the firm decides on equity and debt issuance, and let e_{it-1} denote net equity issuance at the end of period $t-1$. A negative value of e_{it-1} indicates a net dividend payment from the firm to shareholders. Capital in period t is given by:

$$k_{it} = q_{it-1} + e_{it-1} + p^S_{it-1}\tilde{b}^S_{it} + p^L_{it-1}(\tilde{b}^L_{it} - b_{it-1}) - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1}) \tag{16}$$

The stock of firm assets in period t after production and repayment of debt is

$$q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma\tilde{b}^L_{it} + (1-\tau)\left[y_{it} + \varepsilon_{it}k_{it} - w_t l_{it} - \delta k_{it} - f - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})\right]. \tag{17}$$

Definition: Limited Liability. Shareholders are free to default and hand over the firm's assets to creditors for liquidation. A fixed fraction ξ of firm assets is lost in this case.

The timing is summarized in Figure 5. At the end of period $t-1$, a firm has an amount b_{it-1} of long-term debt outstanding and assets q_{it-1}. The firm knows the aggregate state of the economy S_{t-1}, including aggregate productivity z_t. It chooses capital k_{it} by issuing equity e_{it-1} and by selling short-term bonds $\tilde{b}^S_{it}$ and additional long-term bonds $\tilde{b}^L_{it} - b_{it-1}$. In period t, the firm hires labor l_{it} and produces output y_{it}. The idiosyncratic capital quality shock ε_{it} is realized and the firm decides whether to default. If it decides not to default, the firm continues with assets q_{it} and outstanding long-term debt $b_{it} = (1-\gamma)\tilde{b}^L_{it}$.

4.2. Firm Problem

Firms maximize shareholder value and discount cash flows at the international risk-free rate r. Conditional on not defaulting, shareholder value at the end of period $t-1$ can be written as the sum of assets in place and a term which depends on future firm behavior: $q_{it-1} + V_{t-1}(b_{it-1}, S_{t-1})$. Because there are no equity adjustment costs, the amount of assets in place, q_{it-1}, has no influence on the optimal firm policy and the value $V_{t-1}(b_{it-1}, S_{t-1})$.

The amount of assets after production q_{it} in (17) is an increasing function of ε_{it}. There is a unique threshold realization $\bar{\varepsilon}_{it}$ which sets shareholder value to zero:

$$\bar{\varepsilon}_{it}: \quad q_{it} + \mathbb{E}_{S_t|S_{t-1}} V_t\left((1-\gamma)\tilde{b}^L_{it}, S_t\right) = 0 \tag{18}$$

If ε_{it} is smaller than $\bar{\varepsilon}_{it}$, the firm optimally decides to default.

We assume that the firm has no ability to commit to future actions. This lack of commitment not only affects the firm's default choice, but also its decision of how much to produce and how to finance capital. The firm must therefore take its own future behavior as given. It can influence the value $\mathbb{E}_{S_t|S_{t-1}} V_t((1-\gamma)\tilde{b}^L_{it}, S_t)$ through today's choice of long-term debt $\tilde{b}^L_{it}$.

Prior to the draw of ε_{it}, the firm chooses labor demand l_{it}:

$$l_{it} = \frac{\zeta(1-\psi)y_{it}}{w_t} \quad \Leftrightarrow \quad l_{it} = \left(\frac{\zeta(1-\psi)z_t k_{it}^{\psi\zeta}}{w_t}\right)^{\frac{1}{1-\zeta(1-\psi)}} \tag{19}$$

Finally, we consider the firm's capital choice. At the end of period $t-1$, the firm chooses capital k_t and its financing mix: e_{it-1}, $\tilde{b}^S_{it}$, and $\tilde{b}^L_{it}$. Given a stock of assets in place q_{it-1}, existing debt b_{it-1}, and the aggregate state of the economy S_{t-1}, a firm solves:

$$\max_{\substack{k_{it}, e_{it} \geq \underline{e}, \\ \tilde{b}^S_{it}, \tilde{b}^L_{it}, \bar{\varepsilon}_{it}}} \quad -e_{it-1} + \frac{1}{1+r}\int_{\bar{\varepsilon}_{it}}^{\infty} \left[q_{it} + \mathbb{E}_{S_t|S_{t-1}} V_t\left((1-\gamma)\tilde{b}^L_{it}, S_t\right)\right] \varphi(\varepsilon) d\varepsilon \tag{20}$$

$$\begin{aligned}
\text{subject to:} \quad & q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma\tilde{b}^L_{it} + (1-\tau)[y_{it} + \varepsilon_{it}k_{it} - w_t l_{it} - \delta k_{it} - f - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})] \\
& y_{it} = z_t\left(k_{it}^{\psi} l_{it}^{1-\psi}\right)^{\zeta} \\
& l_{it} = \left(\frac{\zeta(1-\psi)z_t k_{it}^{\psi\zeta}}{w_t}\right)^{\frac{1}{1-\zeta(1-\psi)}} \\
& \bar{\varepsilon}_{it}: \quad q_{it} + \mathbb{E}_{S_t|S_{t-1}} V_t\left((1-\gamma)\tilde{b}^L_{it}, S_t\right) = 0 \\
& k_{it} = q_{it-1} + e_{it-1} + p^S_{it-1}\tilde{b}^S_{it} + p^L_{it-1}(\tilde{b}^L_{it} - b_{it-1}) - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1})
\end{aligned}$$

The firm's choice of e_{it-1} is bounded from below: $e_{it-1} \geq \underline{e}$, with $\underline{e} < 0$. This constitutes an upper limit for dividend payments.[11]

4.3. Creditors' Problem

The optimal firm policy crucially depends on the two bond prices p^S_{it-1} and p^L_{it-1}. Competitive creditors break even on expectation. Like shareholders, they discount cash flows at the international risk-free rate r. In case of default, the value of the firm's assets is

$$\underline{q}_{it} \equiv k_{it} + (1-\tau)(y_{it} + \varepsilon_{it} k_{it} - w_t l_{it} - \delta k_{it} - f). \tag{21}$$

At this point, creditors liquidate the firm's assets and receive $(1-\xi)\underline{q}_{it}$. Because short-term debt and long-term debt have equal seniority, the price of short-term debt is

$$p^S_{it-1} = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon}_{it})](1+c) + \frac{(1-\xi)}{\tilde{b}^S_{it} + \tilde{b}^L_{it}} \int_{-\infty}^{\bar{\varepsilon}} \underline{q}_{it} \varphi(\varepsilon) d\varepsilon\right] \tag{22}$$

The break-even price of short-term debt p^S_{it-1} only depends on firm behavior at time t, in particular on the risk of default $\Phi(\bar{\varepsilon}_{it})$. In contrast, the price of long-term debt p^L_{it-1} also depends on the future market value of long-term debt $p^L_{it} = g_t(\,(1-\gamma)\tilde{b}^L_{it}\,,\, S_t\,)$:

$$p^L_{it-1} = \frac{1}{1+r}\Bigg[[1-\Phi(\bar{\varepsilon}_{it})]\left[\gamma + c + (1-\gamma)\, g_t\left((1-\gamma)\tilde{b}^L_{it}, S_t\right)\right] + \frac{(1-\xi)}{\tilde{b}^S_{it} + \tilde{b}^L_{it}} \int_{-\infty}^{\bar{\varepsilon}} \underline{q}_{it} \varphi(\varepsilon) d\varepsilon\Bigg] \tag{23}$$

Because the future price of long-term debt $p^L_{it} = g_t(\,(1-\gamma)\tilde{b}^L_{it}\,,\, S_t\,)$ depends on future firm behavior, it is a function of the future state of the firm. Because the firm cannot directly control future firm behavior, the only way in which it can influence the future bond price is through today's choice of long-term debt $\tilde{b}^L_{it}$.

4.4. Equilibrium Firm Policy

In equilibrium, a firm maximizes shareholder value (20) subject to creditors' two break-even conditions (22) and (23). When selling long-term debt, a firm would like to commit to maintaining low levels of default risk in the future. This would raise today's price of long-term debt and lower the cost of capital. But such a promise is not credible. Once the debt has been sold, the firm has no incentive to take the effects of its actions on the

[11] If the stock of existing debt b_{it-1} is sufficiently large, the firm may find it optimal to choose a corner solution and pay out the entire asset value of the firm as dividend: $e_{it-1} = -q_{it-1}$. In practice, it is illegal to pay dividends which substantially exceed firm earnings and deplete a firm's stock of capital. We choose the value of the constraint $\underline{e}$ such that it rules out this corner solution but is not binding in equilibrium. The exact value of $\underline{e}$ does not affect equilibrium variables.

value of existing debt into account. Because creditors have rational expectations, they correctly anticipate and price in the firm's future behavior.

Because we assume that the firm has no ability to commit to future actions, it must take its own future behavior as given and chooses today's policy as a best response. In other words, the firm plays a game against its future selves. As in Klein, Krusell, and Rios-Rull (2008), we restrict attention to the Markov Perfect equilibrium, i.e. we consider policy rules which are functions of the payoff-relevant state variables. The time-consistent policy is a fixed point in which the future firm policy coincides with today's firm policy.

The value $V_{t-1}(b_{it-1}, S_{t-1})$ can be computed recursively. We define the sum of assets in place q_{it-1} and equity issuance e_{it-1} as a choice variable: $\tilde{e}_{it-1} \equiv q_{it-1} + e_{it-1}$. Each period, the firm chooses a policy vector $\phi(b, S) = \{k, \tilde{e}, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$ which solves

$$V(b, S) = \max_{\phi(b,S)=\left\{\substack{k,\tilde{e}\geq\underline{\tilde{e}},\\ \tilde{b}^S,\tilde{b}^L,\bar{\varepsilon}}\right\}} -\tilde{e} + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty}\left[q' + \mathbb{E}_{S'|S}V\left((1-\gamma)\tilde{b}^L, S'\right)\right]\varphi(\varepsilon)d\varepsilon \tag{24}$$

$$\begin{aligned}
\text{s.t.:}\quad & q' = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\left[y + \varepsilon k - w(S)l - \delta k - f - c(\tilde{b}^S + \tilde{b}^L)\right] \\
& y = z'\left(k^{\psi}l^{1-\psi}\right)^{\zeta} \\
& l = \left(\frac{\zeta(1-\psi)z'k^{\psi\zeta}}{w(S)}\right)^{\frac{1}{1-\zeta(1-\psi)}} \\
& \bar{\varepsilon}:\quad q' + \mathbb{E}_{S'|S}V\left((1-\gamma)\tilde{b}^L, S'\right) = 0 \\
& k = \tilde{e} + p^S\tilde{b}^S + p^L(\tilde{b}^L - b) - H(\tilde{b}^S, \tilde{b}^L, b) \\
& p^S = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\ \varphi(\varepsilon)d\varepsilon\right] \\
& p^L = g(b, S) = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\left[\gamma + c + (1-\gamma)\,g\left((1-\gamma)\tilde{b}^L, S'\right)\right]\right. \\
& \qquad\qquad\qquad\qquad\qquad\qquad \left. + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\ \varphi(\varepsilon)d\varepsilon\right]
\end{aligned}$$

Firm outcomes differ ex-post because of the i.i.d. shock ε. However, because there are no equity adjustment costs, past earnings do not affect the current optimal firm policy $\phi(b, S)$.

To construct aggregate variables, we assume a constant unit mass of firms. Defaulting firms exit the economy and are replaced by new entrants. To enter, firms pay a cost which is financed with long-term debt. This entry cost is set such that entrants always operate with the same amount of outstanding long-term debt b as incumbent firms. This assumption ensures that at any point in time the mass of firms remains constant and that all firms in the economy are ex-ante identical.[12]

[12] Crouzet (2017), Karabarbounis and Macnamara (2019), and Jungherr and Schott (2020) study heterogeneous firm models with risky long-term debt without aggregate shocks.

4.5. Households

We close the model by introducing a representative domestic household. The household works, consumes, and invests its savings at the international risk-free rate r. Government revenue from taxation is paid out to the household as a lump-sum transfer. We assume GHH preferences over consumption C and labor L. Period utility is therefore

$$u\left(C - \frac{L^{1+\theta}}{1+\theta}\right) , \tag{25}$$

with $u(\cdot)$ being strictly increasing and concave, and $\theta > 0$. These preferences yield the labor supply curve:

$$w_t = L_t^{\theta} \tag{26}$$

4.6. Equilibrium

We study the equilibrium of a dynamic open economy business cycle model with a given international risk-free rate r and an endogenous wage w. The aggregate state of the economy consists of aggregate productivity z' and the aggregate stock of existing debt B: $S = (z', B)$.

Definition: Recursive Competitive Equilibrium. A recursive competitive equilibrium consists of *(i)* a policy vector $\phi(b, S) = \{k, \tilde{e}, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$, bond prices p^S and p^L, and a value function $V(b, S)$, *(ii)* a wage function $w(S)$, and *(iii)* a stochastic aggregate law of motion $S' = F(S)$ such that:

1. $\phi(b, S)$, p^S, p^L, and $V(b, S)$ solve the firm problem (24) for $b = B$
2. The labor market clears:
$$L(S) = w(S)^{\frac{1}{\theta}} = l(b, S) \quad \text{for } b = B$$
3. The stochastic aggregate law of motion $S' = F(S)$ is consistent with individual behavior:
$$B' = (1-\gamma)\tilde{b}^L(b, S) \quad \text{for } b = B$$

GDP in this economy is

$$Y = y - f - \xi \int_{-\infty}^{\bar{\varepsilon}} \underline{q}\, \varphi(\varepsilon)\, d\varepsilon - H(\tilde{b}^{S'}, \tilde{b}^{L'}, b') \tag{27}$$

4.7. Solution Method

We find the global solution to the dynamic firm problem in (24) and the equilibrium defined above using value function iteration and interpolation. The key difficulty consists in finding the equilibrium price of long-term debt p^L. Optimal firm behavior depends on p^L which itself depends on the expected future price of long-term debt which in turn depends on future firm behavior. We solve this fixed point problem by computing the equilibrium of a finite-horizon economy. Starting from a final date, we iterate backward until all prices and quantities have converged. We then use the first-period equilibrium allocation as the equilibrium of the infinite-horizon economy. This means that we iterate simultaneously on the value $V(b, S)$ and the long-term bond price p^L (as in Hatchondo and Martinez, 2009). The presence of the idiosyncratic i.i.d. capital quality shock ε with continuous probability distribution $\varphi(\varepsilon)$ facilitates the computation of p^L (*cf.* Chatterjee and Eyigungor, 2012).

In solving (24), firms take the wage rate $w(S)$ and the aggregate law of motion $S' = F(S)$ as given. To compute these equilibrium objects, we start with a guess for $w(S)$ and a candidate law of motion $S' = F(S)$. We then solve the firm problem (24) and use its solution to update the aggregate law of motion $S' = F(S)$. Once individual firm behavior is consistent with the aggregate law of motion, we use labor demand $l(b, S)$ with $b = B$ and labor supply $L(S)$ to update our initial guess for the wage function $w(S)$. See Appendix C.1 for further details.

4.8. Calibration

We follow Arellano et al. (2019) in calibrating an open economy business cycle model to US data. Mendoza and Quadrini (2010) document that about one half of the rise in net borrowing by the US non-financial sector since the mid-1980s has been financed by net foreign capital inflows, and by 2008 about one half of the stock of Treasury bills held outside the US financial sector was owned by foreign agents.[13]

For some of the model parameters we use standard values while others are calibrated to match moments from the US non-financial corporate sector. All parameters are summarized in Table 1.

The model period is one year. The international riskless rate is set to $r = 1/0.97 - 1 = 3.09\%$. The debt coupon is $c = r$ which implies that the equilibrium price of a riskless short-term bond and a riskless long-term bond are both equal to one.

The production technology parameters ζ and ψ are taken from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The annual depreciation rate δ is 10%. The fixed cost f is set to generate zero firm profits on average which implies that $\mathbb{E}\, V(0, z) = 0$. The value for θ is chosen to generate a Frisch elasticity of labor supply of four as in King and Rebelo (1999).

The probability distribution of the idiosyncratic capital quality shock ε is assumed to be Normal with zero mean and standard deviation σ_ε. The natural logarithm of

[13]The main advantage of working with an open economy is that this setup avoids the strongly procyclical riskless real interest rate characteristic of the standard closed economy RBC model but at odds with the US post-war data. Alternative approaches include habits (Beaudry and Guay, 1996; Winberry, 2016) or nominal rigidities in combination with an appropriately specified monetary policy rule (e.g. Bernanke et al., 1999).

Table 1: Parametrization

Parameter	Description	Value
r	riskless rate	0.0309
c	debt coupon	r
ζ	technology parameter	0.75
ψ	technology parameter	0.33
δ	depreciation rate	0.1
f	fixed cost	0.1641
θ	inverse Frisch elasticity	0.25
ρ_z	persistence aggregate shock	0.909
σ_z	st. dev. aggregate shock	0.0028
τ	corporate income tax rate	0.4
γ	repayment rate long-term debt	0.1284
σ_ε	st. dev. idiosyncratic shock	0.652
ξ	default cost	0.669
η	debt issuance cost	0.0077

aggregate productivity follows an AR(1) process:

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t, \tag{28}$$

where ϵ_t is white noise with standard deviation σ_z. The autocorrelation ρ_z is 0.909, as in Khan and Thomas (2013). The standard deviation σ_z generates the same volatility of GDP as in the US data.

We follow Gomes et al. (2016) in setting the tax rate $\tau = 0.4$.[14] The repayment rate of long-term debt γ is a key parameter. We set it to match the Macaulay duration of US corporate bonds with remaining term to maturity above one year. Gilchrist and Zakrajsek (2012) calculate an average duration of 6.47 years. This implies $\gamma = 0.1284$.

The remaining three parameters σ_ε, ξ, and η are chosen to match moments from the US non-financial corporate sector: firm leverage, the average credit spread, and the share of long-term debt. The leverage ratio is informative about the standard deviation of the firm-specific capital quality shock σ_ε, as a higher volatility of earnings induces firms to reduce leverage in order to contain the risk of default. The average credit spread pins down the default cost ξ. The share of long-term debt is informative about the debt issuance cost η, as a higher issuance cost increases firms' long-term debt share in the model.[15]

[14] Hennessy and Whited (2005) suggest a value of 0.3. Gomes et al. (2016) argue that τ should be thought of as capturing additional relative benefits of using debt rather than equity (e.g. equity issuance costs).

[15] Appendix C.3 provides results on the sensitivity of key model moments with respect to parameter values.

Table 2: Model Fit

Moment	Data	Model
Leverage: Firm debt / Firm assets	29.3%	29.3%
Long-term debt share	75.4%	75.4%
Average credit spread	2.3%	2.3%

Note: Data on leverage and the long-term debt share is from Compustat. The average credit spread is from Adrian et al. (2013). See Appendix C.2 for details.

Table 2 compares the empirical moments to simulated data from the business cycle model. The model matches the data very well. Our calibration is also broadly consistent with a number of untargeted moments. Altınkılıç and Hansen (2000) provide micro-evidence for the debt issuance cost η. They calculate an average underwriter spread of 1.1% of bond proceeds. Our model generates a value of 0.8%. The annual default rate in our model is 2.6%, which is slightly lower than the estimate for the business failure rate used in Bernanke et al. (1999) or the average of Moody's expected default frequency across rated and unrated Compustat firms reported in Hovakimian, Kayhan, and Titman (2011).

4.9. Quantitative Results

We use the numerical solution of our dynamic business cycle model to study the economy's response to cyclical shocks. We show that the model replicates the dynamic co-movement between firm debt and output growth. Slow debt gives rise to deep recessions and slow recoveries.

4.9.1. Slow Debt

Figure 6 shows the co-movement between firm credit and output. The first four blue bars on the left show the US data. They represent pairwise correlations between growth in firm debt in year t and GDP growth in year $t + x$. They are identical to the bars on the left hand side of Figure 2. The second group of green bars, labeled *LTD Model*, shows the corresponding moments generated from simulated time series of the dynamic business cycle model with short-term and long-term debt described above.

An important result is that the model generates the slow adjustment of debt. The model correlations tend to be slightly lower than in the data, but the overall pattern of the empirical correlations is well captured. In particular, the model correlations display the characteristic peak at the first lag of output observed in the data. These moments were not targeted during the calibration of the model.

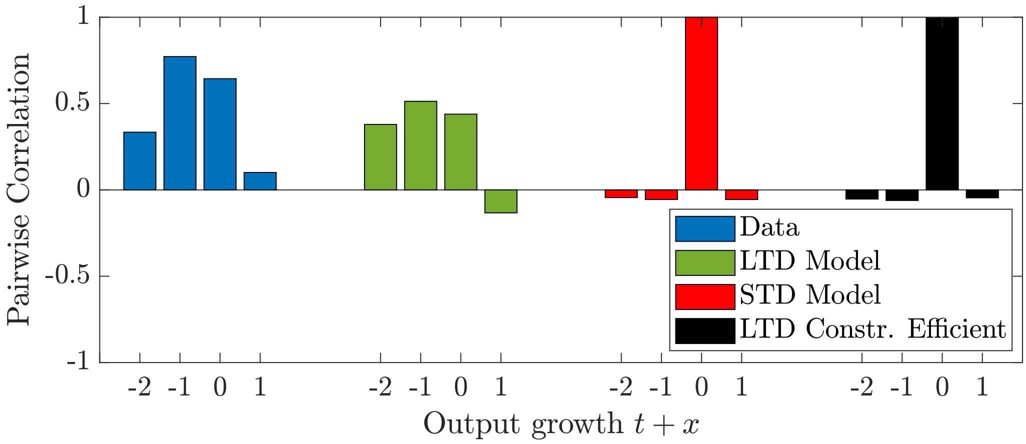


Figure 6: Correlations Firm Credit Growth t and Output Growth $t + x$

Note: Bars show pairwise correlations between annual growth of total firm debt at the end of year t and GDP growth in year $t+x$. The four blue bars to the left (*Data*) are calculated using Flow of Funds data on real total debt of non-financial firms and real GDP (*cf.* Figure 2). All other correlations are calculated from simulated model data.

4.9.2. Deep Recessions

We now explore the economic mechanism which allows the model to replicate the slow-moving behavior of firm debt. Figure 7 shows impulse response functions following a negative shock to aggregate productivity z_t of the size of 2.4 standard deviations at $t = 1$. The solid green line depicts the economy's response in the model described above (*LTD Model*). The dash-dotted yellow line shows the response of output, capital, labor, and wages in a frictionless model without default costs, taxes, or debt issuance costs (*FL Model*). The Modigliani-Miller irrelevance result holds in the frictionless model.[16]

In both models, firms react to a negative shock to z_t by reducing investment and labor demand. In the long-term debt model, firms enter the downturn with an existing stock of long-term debt which has been issued prior to the shock, when investment and debt issuance were high. The fall in investment increases the ratio of existing debt b over capital k. As shown above, firms do not internalize potential default costs which accrue to the holders of existing debt. When the ratio of existing debt over capital rises, firms choose higher levels of leverage and default risk. Leverage is increased by reducing total debt at a slower rate than capital. This mechanism generates the slow response of total debt relative to capital and output shown in Figure 7.[17]

The increase in leverage also maps into a rise in credit spreads during the downturn. Firms issue short- and long-term debt. The credit spread on short-term debt (*ST Spread*, green solid line) rises by more during the downturn than the spread on long-term debt (*LT Spread*, green dashed line). This is because the price of short-term debt p^S only depends on the risk of default next period, while the price of long-term debt p^L depends

[16]See Appendix C.4 for a detailed description of the frictionless model.

[17]A linear debt issuance cost such as (15) can generate an inaction region for debt issuance. This does not play a role in our model. Given our parametrization, firms issue positive amounts of short-term debt and long-term debt in each period.

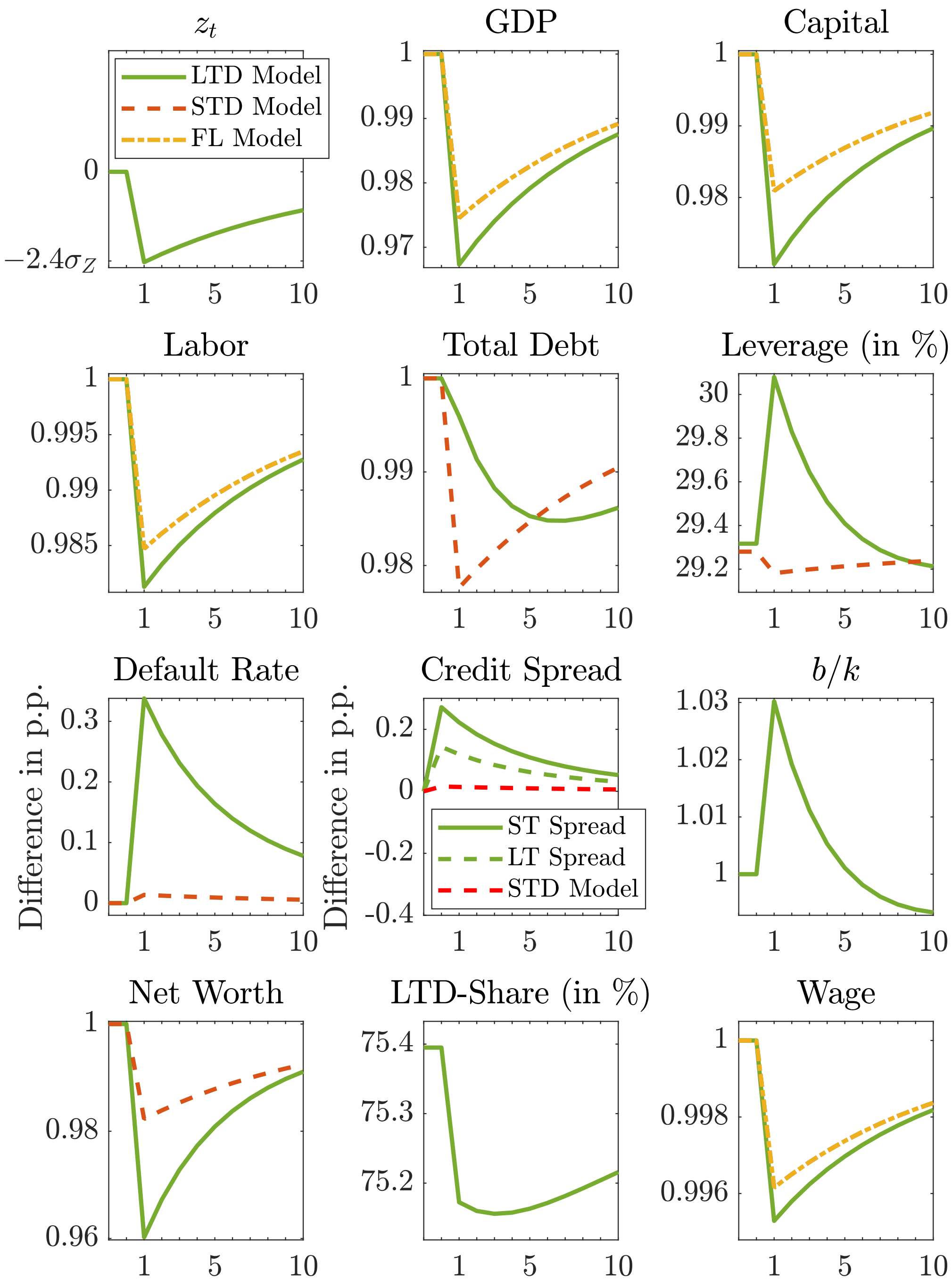


Figure 7: Impulse Response Functions

LTD Model: Green lines show impulse response functions in the benchmark model with long-term and short-term debt (Section 4.6). In the *Credit Spread* panel, the spread on short-term debt in the benchmark model is shown by the solid green line (*ST Spread*), the spread on long-term debt is the dashed green line (*LT Spread*). *STD Model:* Red dashed lines show impulse response functions in a short-term debt model (Appendix C.5). *FL Model:* Yellow dash-dotted lines show impulse response functions of GDP, capital, labor, and the wage rate in a frictionless model (Appendix C.4).

on expected default rates in all future periods.

Within a period, the initial fall in capital and the corresponding rise in default risk amplify each other. Higher default risk increases firms' cost of capital through higher credit spreads and thereby further discourages investment. Capital falls by more, which in turn raises the ratio b/k. Firms respond by choosing even higher levels of leverage and default risk which again drives up the default risk and credit spreads and so forth. This adverse feedback loop between high default risk and low investment explains the difference between the initial output response of the long-term debt model and the frictionless model. Instead of a drop in output of 2.5 percentage points as in the frictionless model, GDP falls on impact by 3.25 percentage points in the long-term debt model. The unconditional volatility of output is increased by 20% relative to the frictionless model.[18]

4.9.3. Slow Recoveries

We showed how slow debt gives rise to an amplified output response following a shock. Figure 7 shows that the long-term debt model also generates a slow recovery.[19] Several periods after the shock, the distance of output from its unconditional mean is still larger relative to its frictionless counterpart. With the aggregate stock of existing debt being the only endogenous state variable of the economy, these dynamics are shaped by firms' endogenous adjustment of the stock of existing debt b.

After the initial shock, the ratio b/k, leverage, the default rate, and credit spreads remain elevated for several periods. Firms understand that by actively reducing long-term debt, they can decrease the future stock of existing debt and thereby also future levels of b/k, leverage, and default risk. Long-term debt can be decreased in two ways: 1.) Firms can reduce total debt for a given long-term debt share, or 2.) firms can reduce the long-term debt share for a given level of total debt. In the first case, firms forgo the tax benefit of debt. In the second case, firms incur higher future costs of debt issuance. While firms fully internalize these costs, they do not internalize all benefits of lower future default risk. Most of these benefits accrue to the holders of existing long-term debt. For this reason, firms choose to adjust the stock of long-term debt slowly. This implies that b/k, leverage, the default rate, and credit spreads remain elevated after the initial shock. Through this persistent increase in the cost of capital, slow debt generates slow recoveries.

These results highlight the crucial role of history dependence in firms' debt management. The issuance of long-term debt during a boom creates a liability which extends well into the subsequent downturn. Firms are reluctant to reduce the high debt levels inherited from the past because the benefits of this reduction would mostly fall to the holders of existing debt. The resulting persistent increase in default risk and credit spreads amplifies and prolongs the downturn.

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[18]The parameter sensitivity analysis in Appendix C.3 provides additional results on the tight link between the slow adjustment of firm debt and GDP volatility in this model.

[19]We use the term 'slow recovery' in the sense of Taylor (2014) and Fernald, Hall, Stock, and Watson (2017). In their terminology, a recovery is 'slow' if output remains below trend for a long time as was the case for the US after the 2008-09 recession.

Table 3: Business Cycle Statistics

Variable x (in %)		**Data**	**LTD Model**	**STD Model**
ΔTotal Debt	$corr(x, \Delta\text{GDP})$	0.59	0.44	1.00
	σ_x	4.22	0.34	0.96
	ρ_x	0.59	0.81	-0.04
Leverage: Debt / Assets	$corr(x, \Delta\text{GDP})$	-0.24	-0.61	0.21
	σ_x	2.32	0.45	0.10
	ρ_x	0.68	0.73	0.91
Default Rate	$corr(x, \Delta\text{GDP})$	-0.48	-0.33	-0.21
	σ_x	0.72	0.25	0.01
	ρ_x	0.37	0.85	0.91
Average Credit Spread	$corr(x, \Delta\text{GDP})$	-0.89	-0.33	-0.21
	σ_x	0.83	0.15	0.02
	ρ_x	0.52	0.84	0.91
Term Structure	$corr(x, \Delta\text{GDP})$	0.57	0.36	-
	σ_x	0.41	0.09	-
	ρ_x	0.51	0.83	-
b/k	$corr(x, \Delta\text{GDP})$	-0.30	-0.60	-
	σ_x	1.92	0.45	-
	ρ_x	0.63	0.76	-
LTD Share	$corr(x, \Delta\text{GDP})$	-0.08	0.01	-
	σ_x	5.31	0.35	-
	ρ_x	0.85	0.97	-

Note: Δ is the real growth rate. Other variables are in levels. $corr(x, \Delta\text{GDP})$ is the contemporaneous correlation between x and real GDP growth. σ_x and ρ_x denote the standard deviation and autocorrelation. *Data sources:* Flow of Funds, Compustat, St. Louis Fed (FRED), and Giesecke, Longstaff, Schaefer, and Strebulaev (2014). All data is annual. See Appendix C.2 for details.

4.9.4. Business Cycle Statistics

Table 3 compares the cyclical behavior of key model variables to the data. The table reports each variable's contemporaneous correlation with GDP growth, as well as its standard deviation and autocorrelation.

The model replicates the counter-cyclical behavior of leverage, the default rate, and credit spreads. This is a success of the model. Financial accelerator models often generate pro-cyclical leverage and default risk in response to standard first moment shocks (e.g. Carlstrom and Fuerst, 1997). Adding second moment shocks results in counter-cyclical default risk and credit spreads, but leverage remains pro-cyclical (e.g. Gilchrist et al., 2014). Our long-term debt model generates counter-cyclical default rates, credit spreads, and leverage.

The model is also successful in generating a pro-cyclical term structure of credit spreads. The term structure of credit spreads is the difference between the credit spread on long-term debt and short-term debt. It decreases during a recession because the short-term spread increases by more than the long-term spread (both in the model and in the data). Whereas the short-term spread only depends on next period's risk of default, the long-term spread is a weighted average of default risk in all future periods and is therefore less sensitive to short-term fluctuations.[20]

In the model, variations in the ratio of existing debt over capital b/k are an important part of the mechanism which generates deep recessions and slow recoveries. Table 3 shows that the model captures the counter-cyclical behavior of this variable together with a similar degree of persistence as in the data. Importantly, the quantitative model does not overstate the empirical volatility of b/k.

Whereas the cyclicality of financial variables in the long-term debt model is broadly in line with the data, the size of these fluctuations is generally too small. This suggests an important role of additional forces (e.g. credit supply shocks, monetary policy shocks) in shaping the empirical behavior of financial variables in the corporate sector.

4.9.5. The Role of Long-Term Debt

To illustrate the crucial role of long-term debt in generating slow debt and deep recessions, we compare the results described above to an alternative model in which firms use only short-term debt (*STD Model*).[21]

Consider the third group of red bars in Figure 6 which shows correlations between growth in firm debt at time t and output at time $t + x$ in the short-term debt model. The difference between the dynamic patterns generated by the *STD Model* and the *LTD Model* is striking. In the short-term debt model, firm debt strongly co-moves with contemporaneous output. In contrast to the long-term debt model and the data, there is no positive correlation with lagged output in the short-term debt model.

[20] This result is consistent with the estimated impulse responses of corporate bond spreads of different maturities to credit shocks in Gilchrist, Yankov, and Zakrajšek (2009).

[21] For the short-term debt model, we set $\gamma = 1$ and calibrate it to the same moments as the benchmark model with long-term debt. See Appendix C.5 for details.

The dashed red lines in Figure 7 show impulse response functions from the short term debt model. The dynamics of GDP, capital, labor, and wages are virtually identical to the frictionless model. Leverage, the default rate, and credit spreads hardly move over the business cycle. The model lacks the feedback loop between default risk and investment which gives rise to amplification in the long-term debt model.

Finally, in the third column of Table 3 we present business cycle statistics from the short-term debt model. The co-movement between output and firm debt is too strong and firm debt is not persistent enough. The lack of volatility in financial variables is even more pronounced than in the long-term debt model.

While the short-term debt model cannot replicate the empirical lag structure between output and credit growth, the absence of a lag in the correlations of Figure 6 is in line with the firm-level evidence from Section 2 where we showed that credit of firms with low long-term debt shares does not lag output growth.

4.10. Constrained Efficiency

In the long-term debt model, firms adjust debt slowly because they do not internalize all associated costs. They exert an externality on the holders of previously issued debt who bear a large part of the costs of increased default risk. Ultimately though, the costs of slow debt fall back on shareholders. Creditors break even on expectation. They correctly anticipate and price in all effects of existing debt on current and future firm behavior.

When a firm sells long-term debt, it would like to promise to maintain low future levels of default risk in order to raise today's price of long-term debt. But because a low bond price today becomes a sunk cost tomorrow, this promise is not credible. Slow debt is a symptom of firms' lack of commitment to future actions.

A social planner who is subject to the same lack of commitment and faces the same set of constraints as the firm can do better in the sense that the payoff of creditors *and* shareholders can be increased. To see this, we revisit the problem of constrained efficiency studied in the two-period model above. Just as in Section 3.5, we study an individual firm that is controlled by a planner who maximizes the entire firm value, that is, the sum of all equity and debt claims, existing and newly issued bonds alike. The planner solves:[22]

$$W(b,S) = \max_{\left\{k,\tilde{e}\geq\tilde{\underline{e}},\tilde{b}^S,\tilde{b}^L,\bar{\varepsilon}\right\}} \quad p^L b - T(b,S) - \tilde{e} + \frac{1}{1+r}\mathbb{E}_{S'|S}\left\{\int_{\bar{\varepsilon}}^{\infty}\left[q' + W\left((1-\gamma)\tilde{b}^L, S'\right)\right]\varphi(\varepsilon)d\varepsilon\right\} \quad (29)$$

subject to the same set of constraints as in (24). The state-contingent tax $T(b,S)$ in

[22] In order to highlight the commitment problem at the firm level and to stay as close as possible to the equilibrium allocation studied above, the planner chooses the policy of an individual firm without taking into account general equilibrium effects, e.g. on the wage rate.

(29) is specified such that in equilibrium $T(b,S) = p^L b$. In this way, the planner takes into account how firm behavior affects the market value of existing debt, $p^L b$, without mechanically affecting the value $W(b,S)$. The value $W(b,S)$ will differ from $V(b,S)$ in (24) only because of different firm behavior. See Appendix C.6 for details.

Figure 8 compares the solution of the constrained efficient problem in (29) (black dashed lines) to the decentralized long-term debt model (green solid lines). The main result is that neither slow debt nor deep recessions are part of the constrained efficient allocation. The planner strongly reduces total debt in response to a negative productivity shock and thereby avoids the rise in leverage and credit spreads observed in the long-term debt model. This reduces the initial output drop of 3.25 percentage points in the long-term debt model to 2.5 percentage points in the constrained efficient allocation.

The reason for the difference between the constrained efficient allocation and the decentralized long-term debt model is that the planner does not face the same commitment problem as firms. The planner always internalizes the effect of current debt issuance on the payoff of existing creditors. The reason is not that the planner has more commitment power than firms but simply that the planner's objective includes the payoff of creditors *and* shareholders. After a negative shock, slow adjustment of debt would increase the default risk and hurt the holders of existing debt. Because the planner internalizes these costs, debt is adjusted immediately.[23]

The four black bars on the right of Figure 6 show the dynamic correlation between debt and output growth in the constrained efficient solution. Slow debt is absent because the constrained efficient debt issuance policy does not feature history dependence.

5. Conclusion

Firm-level data suggests that the slow adjustment of firm debt to changes in economic activity is related to firms' use of long-term debt. We have shown that introducing long-term debt into a standard business cycle model of production, firm financing, and costly default successfully replicates the empirical lag structure between firm debt and output. The model is also successful in generating counter-cyclical firm leverage, default rates, and credit spreads, as well as the pro-cyclical term structure of credit spreads.

Rising credit spreads during downturns increase firms' cost of capital and thereby amplify the fall in investment and output. Because firms choose to reduce debt only gradually, deep recessions are followed by slow recoveries.

In the model, firms exert an externality on the holders of existing debt. A social planner who internalizes the payoff to existing creditors would implement a different allocation. Neither slow debt nor deep recessions are constrained efficient. The results presented above suggest substantial room for welfare improving stabilization policies.

[23] Absent the commitment problem described above, the downside of long-term debt relative to short-term debt disappears. As a result, only long-term debt is issued in the planner's allocation and the optimal maturity choice is always at a corner solution. Therefore the long-term debt share is acyclical in the constrained efficient allocation. Because only long-term debt is issued, only the long-term spread is displayed for the constrained efficient allocation in Figure 8.

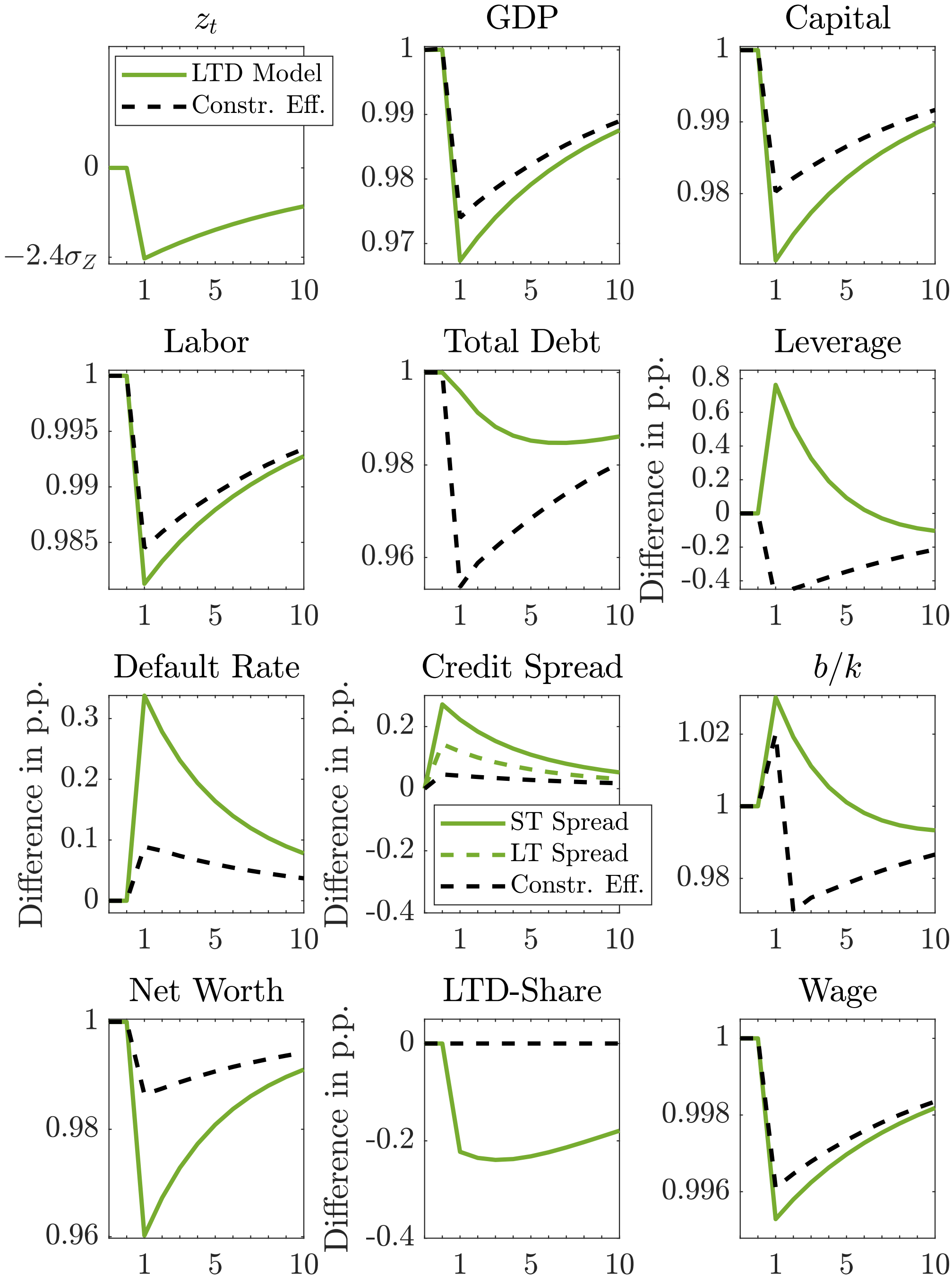

Figure 8: Impulse Response Functions - Constrained Efficiency

LTD Model: Green lines show impulse response functions in the benchmark model with long-term and short-term debt (Section 4.6). In the *Credit Spread* panel, the spread on short-term debt in the benchmark model is shown by the solid green line (*ST Spread*), the spread on long-term debt is the dashed green line (*LT Spread*). *Constr. Eff.:* Black dashed lines show impulse response functions of the constrained efficient allocation (Appendix C.6). Because only long-term debt is issued in the constrained efficient allocation, the *Credit Spread* panel shows only the spread on long-term debt (*Constr. Eff.*).

Studying specific policies which target this inefficiency is an important topic for future work.

Throughout our analysis, aggregate fluctuations were caused by productivity shocks. However, the adverse feedback loop between high credit spreads and low investment triggered by slow debt is a very general mechanism. Any initial change in firm investment induced by various kinds of shocks can be amplified in the same way. It would be interesting to explore the role of different types of shocks (e.g. financial shocks) within the framework described above.

Another potential direction for future work is to add working-capital constraints. When firms need to pay part of the wage bill upfront, credit spreads directly affect firms' labor demand. This generates a time-varying labor wedge which has been found to be important in explaining aggregate fluctuations (e.g. Chari, Kehoe, and McGrattan, 2007; Jermann and Quadrini, 2012; Arellano et al., 2019).

References

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer (2018): "The leverage ratchet effect," *Journal of Finance*, 73, 145–198.

Adrian, T., P. Colla, and H. S. Shin (2013): "Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009," *NBER Macroeconomics Annual*, 27, 159–214.

Aguiar, M., M. Amador, H. Hopenhayn, and I. Werning (2019): "Take the short route: Equilibrium default and debt maturity," *Econometrica*, 87, 423–462.

Altinkiliç, O. and R. S. Hansen (2000): "Are there economies of scale in underwriting fees? Evidence of rising external financing costs," *Review of Financial Studies*, 13, 191–218.

Arellano, C., Y. Bai, and P. J. Kehoe (2019): "Financial frictions and fluctuations in volatility," *Journal of Political Economy*, 127.

Arellano, C. and A. Ramanarayanan (2012): "Default and the maturity structure in sovereign bonds," *Journal of Political Economy*, 120, 187–232.

Beaudry, P. and A. Guay (1996): "What do interest rates reveal about the functioning of real business cycle models?" *Journal of Economic Dynamics and Control*, 20, 1661–1682.

Begley, J. and R. Freedman (2004): "The changing role of accounting numbers in public lending agreements," *Accounting Horizons*, 18, 81–96.

Benmelech, E., N. Kumar, and R. Rajan (2020): "The decline of secured debt," NBER Working Paper No. 26637.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, 1, 1341–1393.

Billett, M. T., T.-H. D. King, and D. C. Mauer (2007): "Growth opportunities and the choice of leverage, debt maturity, and covenants," *Journal of Finance*, 62, 697–730.

Bloom, N., M. Floetotto, N. Jaimovich, I. S. Eksten, and S. J. Terry (2018): "Really uncertain business cycles," *Econometrica*, 86, 1031–1065.

Caggese, A. and A. Perez (2016): "The interaction between household and firm dynamics and the amplification of financial shocks," Working Paper.

Carlstrom, C. T. and T. S. Fuerst (1997): "Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis," *American Economic Review*, 893–910.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007): "Business cycle accounting," *Econometrica*, 75, 781–836.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, indebtedness, and default risk," *American Economic Review*, 102, 2674–2699.

——— (2015): "A seniority arrangement for sovereign debt," *American Economic Review*, 105, 3740–3765.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk shocks," *American Economic Review*, 104, 27–65.

Cooley, T., R. Marimon, and V. Quadrini (2004): "Aggregate consequences of limited contract enforceability," *Journal of Political Economy*, 112, 817–847.

Covas, F. and W. J. Den Haan (2012): "The role of debt and equity finance over the business cycle," *Economic Journal*, 122, 1262–1286.

Crouzet, N. (2017): "Default, debt maturity, and investment dynamics," Working Paper.

DeMarzo, P. and Z. He (2020): "Leverage dynamics without commitment," *Journal of Finance*, forthcoming.

Fernald, J. G., R. E. Hall, J. H. Stock, and M. W. Watson (2017): "The disappointing recovery of output after 2009," NBER Working Paper No. 23543.

Giesecke, K., F. A. Longstaff, S. Schaefer, and I. A. Strebulaev (2014): "Macroeconomic effects of corporate default crisis: A long-term perspective," *Journal of Financial Economics*, 111, 297–310.

Gilchrist, S., J. W. Sim, and E. Zakrajšek (2014): "Uncertainty, financial frictions, and investment dynamics," NBER Working Paper No. 20038.

Gilchrist, S., V. Yankov, and E. Zakrajšek (2009): "Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets," *Journal of Monetary Economics*, 56, 471–493.

Gilchrist, S. and E. Zakrajsek (2012): "Credit spreads and business cycle fluctuations," *American Economic Review*, 102, 1692–1720.

Gomes, J., U. Jermann, and L. Schmid (2016): "Sticky leverage," *American Economic Review*, 106, 3800–3828.

Gomes, J. F., A. Yaron, and L. Zhang (2003): "Asset prices and business cycles with costly external finance," *Review of Economic Dynamics*, 6, 767–788.

Green, D. (2018): "Corporate refinancing, covenants, and the agency cost of debt," Working Paper.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117–125.

Hatchondo, J. C., L. Martinez, and C. Sosa-Padilla (2016): "Debt dilution and sovereign default risk," *Journal of Political Economy*, 124, 1383–1422.

Hennessy, C. A. and T. M. Whited (2005): "Debt dynamics," *Journal of Finance*, 60, 1129–1165.

Hovakimian, A., A. Kayhan, and S. Titman (2011): "Are corporate default probabilities consistent with the static trade-off theory?" *Review of Financial Studies*, 25, 315–340.

Ivashina, V. and B. Vallee (2019): "Weak credit covenants," Working Paper.

Jermann, U. and V. Quadrini (2012): "Macroeconomic effects of financial shocks," *American Economic Review*, 102, 238–271.

Jungherr, J. and I. Schott (2020): "Optimal debt maturity and firm investment," Working Paper.

Karabarbounis, M. and P. Macnamara (2019): "Misallocation and credit market constraints: the role of long-term financing," Working Paper.

Khan, A. and J. K. Thomas (2013): "Credit shocks and aggregate fluctuations in an economy with production heterogeneity," *Journal of Political Economy*, 121, 1055–1107.

King, R. G. and S. T. Rebelo (1999): "Resuscitating real business cycles," *Handbook of Macroeconomics*, 1, 927–1007.

Kiyotaki, N. and J. Moore (1997): "Credit cycles," *Journal of Political Economy*, 105, 211–248.

Klein, P., P. Krusell, and J.-V. Rios-Rull (2008): "Time-consistent public policy," *Review of Economic Studies*, 75, 789–808.

Kopecky, K. A. and R. M. Suen (2010): "Finite state Markov-chain approximations to highly persistent processes," *Review of Economic Dynamics*, 13, 701–714.

Leland, H. (1994): "Bond prices, yield spreads, and optimal capital structure with default risk," Working Paper.

Mendoza, E. G. and V. Quadrini (2010): "Financial globalization, financial crises and contagion," *Journal of Monetary Economics*, 57, 24–39.

Miao, J. and P. Wang (2010): "Credit risk and business cycles," Working Paper.

Nash, R. C., J. M. Netter, and A. B. Poulsen (2003): “Determinants of contractual relations between shareholders and bondholders: investment opportunities and restrictive covenants,” *Journal of Corporate Finance*, 9, 201–232.

Occhino, F. and A. Pescatori (2015): “Debt overhang in a business cycle model,” *European Economic Review*, 73, 58–84.

Paul, P. (2018): “A macroeconomic model with occasional financial crises,” Working Paper.

Poeschl, J. (2018): “Corporate debt maturity and investment over the business cycle,” Working Paper.

Reisel, N. (2014): “On the value of restrictive covenants: Empirical investigation of public bond issues,” *Journal of Corporate Finance*, 27, 251–268.

Roberts, M. R. and A. Sufi (2009): “Control rights and capital structure: An empirical investigation,” *Journal of Finance*, 64, 1657–1695.

Taylor, J. B. (2014): “The role of policy in the Great Recession and the weak recovery,” *American Economic Review*, 104, 61–66.

Winberry, T. (2016): “Lumpy investment, business cycles, and stimulus policy,” Working Paper.

Xiang, H. (2019): “Time inconsistency and financial covenants,” Working Paper.

A. Empirical Facts

In Appendix A.1, we provide additional results on the dynamic co-movement between output and firm credit, and output and firm assets. Details on data sources follow in Appendix A.2.

A.1. Additional Results

Figure 9 repeats the exercise from Figure 2 using quarterly data. The bars to the left of Figure 9 show pairwise correlations between total firm credit growth in quarter t and GDP growth in quarter $t + x$. The bars to the right show the corresponding correlations between corporate debt and corporate value added. The results confirm the slow-moving behavior of debt. The correlations peak at a lag of five to six quarters (firm credit vs. GDP growth) and seven quarters (corporate credit vs. corporate value added), respectively.

For completeness, we also calculate the dynamic co-movement between firm assets and output. Figure 10 shows annual growth rates of real firm assets (book value, marked-to-market) together with annual growth of real GDP. In contrast to firm debt, firm assets do not lag output growth. This is confirmed by Figure 11 which displays pairwise correlations between asset growth (both marked-to-market and at historical cost) and output growth at various time lags.

A.2. Data Sources

The data used in Section 2 is from the Flow of Funds Accounts of the US Federal Reserve Board and from Compustat. Consumer price data comes from the US Bureau of Labor Statistics.

A.2.1. Flow of Funds Data

Annual data retrieved from the Flow of Funds: *GDP* is 'Gross domestic product' (Flow of Funds code FU086902005.A). *Firm Credit* is 'Nonfinancial business; debt securities and loans; liability' (FL144104005.A). *Leverage* is *Firm Credit* divided by the sum of 'Nonfinancial corporate business; total assets' (FL102000005.A) and 'Nonfinancial noncorporate business; total assets' (FL112000005.A). These variables measure assets at book value and marked-to-market. For the book value at historical cost, we use 'Nonfinancial corporate business; total assets at historical cost' (FL102000115.A). *Corporate Credit* is 'Nonfinancial corporate business; debt securities and loans; liability' (FL104104005.A). *Corporate Value Added* is 'Nonfinancial corporate business; gross value added' (FU106902501.A).

Annual flow variables are deflated using the annual 'CPI-All Urban Consumers' from the Bureau of Labor Statistics. End-of-year stock variables are deflated using the seasonally adjusted December value of the 'CPI-All Urban Consumers'.

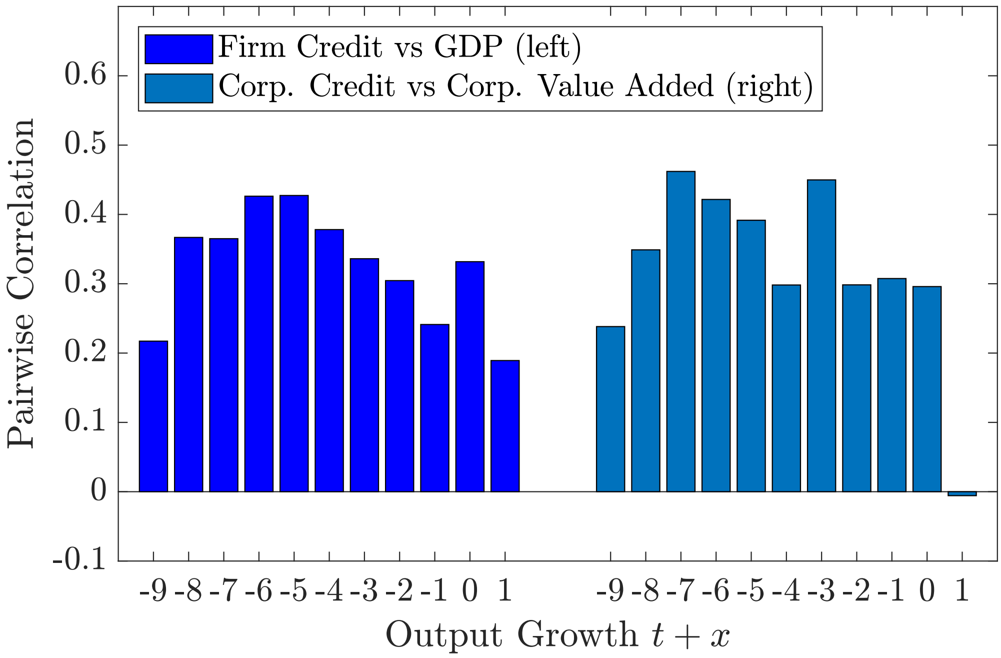

Figure 9: Correlations Firm Credit Growth t with Output Growth $t+x$ (Flow of Funds)
Note: Bars show pairwise correlations. The left bars show correlations between quarterly growth of real total debt of non-financial firms at the end of period t and real GDP growth in period $t+x$. The right bars show correlations between quarterly growth of real total debt of non-financial corporate firms at the end of period t and real growth of non-financial corporate value added in period $t+x$. All variables are seasonally adjusted. Data is from the Flow of Funds 1984-2015.

Figure 9 uses seasonally adjusted quarterly Flow of Funds data. *GDP* is 'Gross domestic product' (FA086902005.Q). *Firm Credit* is 'Nonfinancial business; debt securities and loans; liability' (FA144104005.Q). *Corporate Credit* is 'Nonfinancial corporate business; debt securities and loans; liability' (FA104104005.Q). *Corporate Value Added* is 'Nonfinancial corporate business; gross value added' (FA106902501.Q).

Quarterly flow variables are deflated using the quarter average of the seasonally adjusted monthly 'CPI-All Urban Consumers'. End-of-quarter stock variables are deflated using the end-of-quarter value of the seasonally adjusted monthly 'CPI-All Urban Consumers'.

A.2.2. Compustat Data

We use firm-level data from Compustat 1984-2015. To facilitate comparison with the Flow of Funds data, we only include Compustat firm-year observations which are reported in December of a given year. We also exclude financial firms (SIC codes 6000-6999) as well as firm-year observations with an ISO Currency Code different from US Dollar. Furthermore, we exclude observations with negative *Firm Debt* (annual data item number 34 + data item 9) or *Sales* (data item 12), and those that do not report

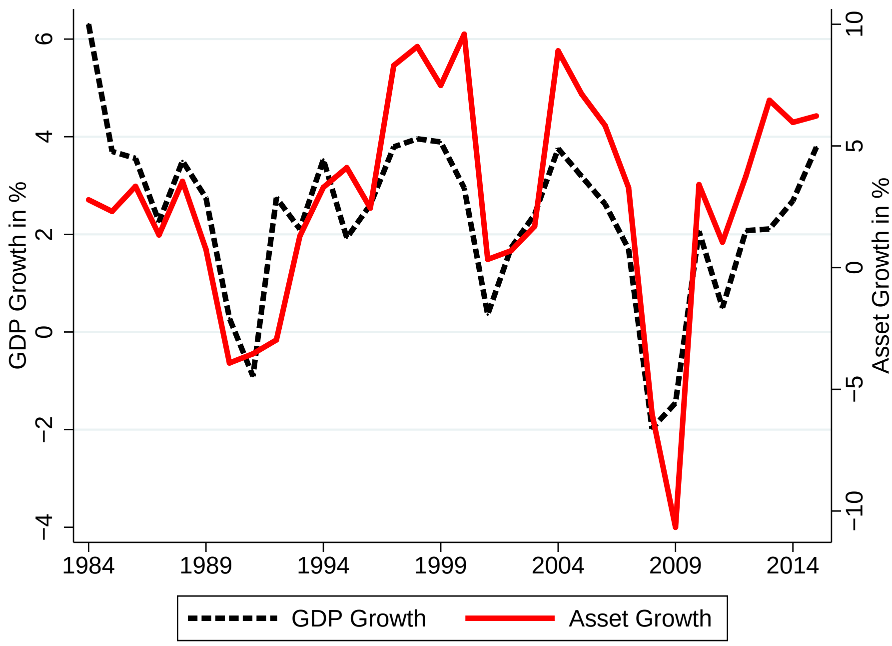

Figure 10: Firm Asset Growth and GDP

Note: Asset Growth (solid red line, right axis) is annual growth of end-of-year real total assets (book value, marked-to-market) of non-financial firms. *GDP Growth* (dashed black line, left axis) is annual growth of real GDP. Data comes from the Flow of Funds.

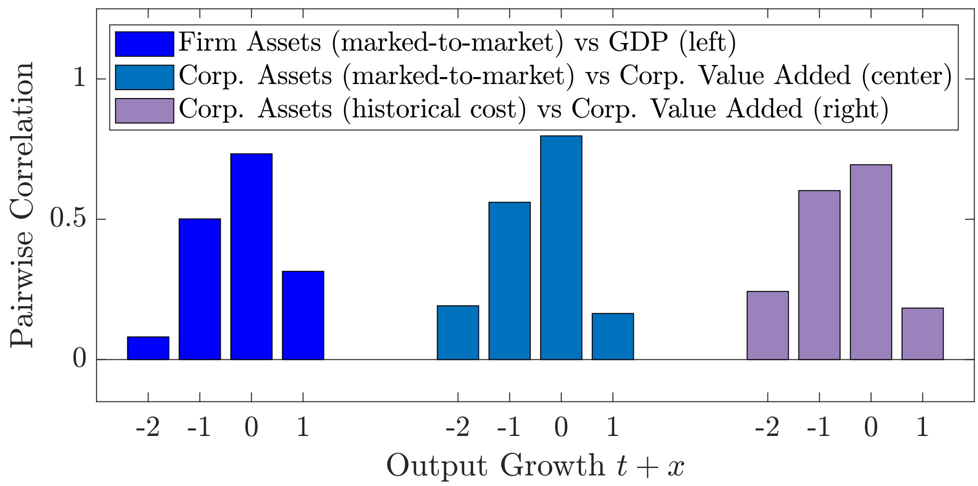

Figure 11: Correlations Firm Asset Growth t with Output Growth $t+x$ (Flow of Funds)

Note: Bars show pairwise correlations. The left bars show correlations between annual growth of real total assets (book value, marked-to-market) of non-financial firms at the end of year t and real GDP growth in year $t+x$. The bars in the center show correlations between annual growth of real total assets (book value, marked-to-market) of non-financial corporate firms at the end of year t and real growth of non-financial corporate value added in year $t+x$. The bars to the right show the corresponding values for real total assets measured at historical cost instead of marked-to-market. Data comes from the Flow of Funds 1984-2015.

Long-term Debt (data item 9), *Firm Debt*, or *Sales*. The *Long-term Debt Share* is *Long-term Debt* (data item 9) divided by *Firm Debt* (data item 34 + data item 9). Total *Firm Debt* in our Compustat sample is on average about 90% of non-financial corporate debt from the Flow of Funds.

B. Two-period Model: Proofs and Derivations

In the two-period model, the firm solves (6) subject to creditors' break-even constraint (8). As shown below, this problem can be re-written in terms of only two choice variables: k and $\bar{\varepsilon}$.

The first step is to express output net of wage payments $y - wl$ as a function of capital only. Given the wage rate w, a necessary and sufficient condition for optimal labor demand is

$$l^* = \frac{\zeta(1-\psi)y}{w} \quad \Leftrightarrow \quad l^* = \left(\frac{\zeta(1-\psi)zk^{\psi\zeta}}{w}\right)^{\frac{1}{1-\zeta(1-\psi)}} \tag{B.30}$$

This implies for output net of wage payments:

$$y^* - wl^* = Ak^{\alpha}, \tag{B.31}$$

where A and α are functions of productivity z, the wage w, and the technology parameters ζ and ψ:

$$A \equiv z^{\frac{1}{1-\zeta(1-\psi)}}[1-\zeta(1-\psi)]\left(\frac{\zeta(1-\psi)}{w}\right)^{\frac{\zeta(1-\psi)}{1-\zeta(1-\psi)}} \quad \text{and} \quad \alpha \equiv \frac{\zeta\psi}{1-\zeta(1-\psi)}, \tag{B.32}$$

with $\alpha \in (0,1)$. Applying (B.31) to the definition of $\bar{\varepsilon}$ in (5) yields

$$\tilde{b}[1+(1-\tau)c] = k+(1-\tau)(Ak^{\alpha}+\bar{\varepsilon}k-\delta k) \Leftrightarrow \tilde{b} = \frac{k+(1-\tau)(Ak^{\alpha}+\bar{\varepsilon}k-\delta k)}{1+(1-\tau)c}. \tag{B.33}$$

Substituting (B.31), (B.33), and (3) into the firm problem in (6), we obtain

$$\begin{aligned} \max_{k,\tilde{b},\bar{\varepsilon},p} \quad & q - k + p(\tilde{b}-b) + \frac{1-\tau}{1+r}k\int_{\bar{\varepsilon}}^{\infty}(\varepsilon-\bar{\varepsilon})\varphi(\varepsilon)d\varepsilon \\ \text{subject to:} \quad & p = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\;\varphi(\varepsilon)d\varepsilon\right] \\ & \tilde{b} = \frac{k+(1-\tau)(Ak^{\alpha}+\bar{\varepsilon}k-\delta k)}{1+(1-\tau)c} \end{aligned} \tag{B.34}$$

The firm problem (B.34) characterizes the equilibrium allocation in terms of the two choice variables k and $\bar{\varepsilon}$. The first-order conditions for capital (10) and $\bar{\varepsilon}$ (11) are derived under the assumption made in the main text that $\xi = 1$. The marginal product of capital is MPK $\equiv A\alpha k^{\alpha-1}$ which is increasing in z (through A) and decreasing in k.

The partial derivative used in (11) is $\partial\tilde{b}/\partial\bar{\varepsilon} = (1-\tau)k/[1+(1-\tau)c]$.

Proof of Proposition 1

Proof. The first order condition for $\bar{\varepsilon}$ is given by (11). At an interior solution, the second derivative of the objective function (B.34) w.r.t. $\bar{\varepsilon}$ is negative. The left-hand side of (11) is increasing in b, with $d/db = \varphi(\bar{\varepsilon})(1+c)$. It follows that with a higher b, (11) can only hold at a higher level of $\bar{\varepsilon}$. For a given k this implies a higher level of $\tilde{b}$ because from (B.33) we have that $\partial\tilde{b}/\partial\bar{\varepsilon} > 0$. It follows that $\partial\tilde{b}/\partial b > 0$. □

Proof of Proposition 2

Proof. By writing (B.33) as

$$\tilde{b} = k\frac{1+(1-\tau)(Ak^{\alpha-1}+\bar{\varepsilon}-\delta)}{1+(1-\tau)c}, \tag{B.35}$$

and dividing the first order condition for $\bar{\varepsilon}$ in (11) by k, we obtain:

$$[1-\Phi(\bar{\varepsilon})]\frac{(1-\tau)}{1+(1-\tau)c}\tau c - \varphi(\bar{\varepsilon})(1+c)\left(\frac{1+(1-\tau)\left(Ak^{\alpha-1}-\delta+\bar{\varepsilon}\right)}{1+(1-\tau)c} - \frac{b}{k}\right) = 0 \tag{B.36}$$

Productivity z enters the firm problem through the term A which is increasing in z. Furthermore, α converges to one when ζ tends towards one. There are two cases to consider: (1.) $b = 0$, and (2.) $b > 0$.

1. If $b = 0$, (B.36) reads as

   $$[1-\Phi(\bar{\varepsilon})]\frac{(1-\tau)}{1+(1-\tau)c}\tau c - \varphi(\bar{\varepsilon})(1+c)\left(\frac{1+(1-\tau)\left(Ak^{\alpha-1}-\delta+\bar{\varepsilon}\right)}{1+(1-\tau)c}\right) = 0 \tag{B.37}$$

   By isolating the *marginal* product of capital MPK $= A\alpha k^{\alpha-1}$ in (10) and combining it with (B.37) to substitute out the *average* product of capital $Ak^{\alpha-1}$, one is left with a single equation with $\bar{\varepsilon}$ as the only endogenous variable. Neither profitability A nor capital k appear in this equation. It follows that the optimal choice of $\bar{\varepsilon}$ and the default rate $\Phi(\bar{\varepsilon})$ do not depend on A if $b = 0$. This result extends to the bond price and the credit spread. From (B.35), leverage is

   $$\frac{\tilde{b}}{k} = \frac{1+(1-\tau)\left(Ak^{\alpha-1}-\delta+\bar{\varepsilon}\right)}{1+(1-\tau)c}. \tag{B.38}$$

   We know from (B.37) that the average product of capital $Ak^{\alpha-1}$ must be constant if $\bar{\varepsilon}$ does not change. It follows that also leverage is constant in A if $b = 0$.

2. If $b > 0$, the first order condition for ε is given by (B.36). The left hand side is obtained by adding the term $\varphi(\bar{\varepsilon})(1+c)b/k$ to the left hand side of (B.37). This

term is positive and decreasing in k:

$$\frac{\partial}{\partial k}\left[\varphi(\bar{\varepsilon})(1+c)\frac{b}{k}\right] = -\frac{b}{k^2}\varphi(\bar{\varepsilon})(1+c) \tag{B.39}$$

If the optimal choice of k is increasing in A, it follows that higher values of A imply lower optimal values of $\bar{\varepsilon}$ if $b > 0$. In this case, the default rate $\Phi(\bar{\varepsilon})$ is falling in A. This implies that the bond price p rises and the credit spread falls in A.

It remains to be shown that also leverage decreases in A if α is sufficiently close to one. From (B.38), leverage is increasing in the term $Ak^{\alpha-1} + \bar{\varepsilon}$. Consider the third term on the left hand side of (10): the *Marginal increase in expected stock of equity* q. This term increases if $\bar{\varepsilon}$ falls. Because $\bar{\varepsilon}$ falls if A rises, this term increases in A. In order for the first order condition (10) to hold, the second term, the *Marginal increase in market value of newly issued debt* $p(\tilde{b} - b)$, must fall if A increases. But the probability $[1 - \Phi(\bar{\varepsilon})]$ increases in A. It follows that the term MPK+$\bar{\varepsilon} = A\alpha k^{\alpha-1} + \bar{\varepsilon}$ must fall in A. If α is sufficiently close to one, this term is approximately equal to the term $Ak^{\alpha-1} + \bar{\varepsilon}$. But if $Ak^{\alpha-1} + \bar{\varepsilon}$ falls in A, also leverage falls in A.

□

Proof of Proposition 3

Proof. Consider the marginal effect (ME_k) of capital on the firm's objective (B.34) as given by the left hand side of first order condition (10):

$$ME_k \equiv -1 + \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1+(1-\tau)\left(A\alpha k^{\alpha-1} - \delta + \bar{\varepsilon}\right)}{1+(1-\tau)c} + \frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}(\varepsilon - \bar{\varepsilon})\varphi(\varepsilon)d\varepsilon = 0 \tag{B.40}$$

Consider an increase in z. This increases A and therefore the marginal product of capital $A\alpha k^{\alpha-1}$. This is the direct effect of an increase in z on the firm's choice k. In addition, there is also an indirect effect of A through $\bar{\varepsilon}$:

$$\frac{\partial ME_k}{\partial A} = \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+(1-\tau)c}\alpha k^{\alpha-1} + \frac{\partial \bar{\varepsilon}}{\partial A}\frac{\partial ME_k}{\partial \bar{\varepsilon}} \tag{B.41}$$

The first term on the right hand side of (B.41) is the direct effect of an increase in A. It is always positive. The second term is the indirect effect which runs through $\bar{\varepsilon}$. From Proposition 2, we know that $\bar{\varepsilon}$ decreases in A if k increases and $b > 0$: $\partial\bar{\varepsilon}/\partial A < 0$. It follows that a fall in $\bar{\varepsilon}$ amplifies the positive response of k to an increase in A if and only if: $\partial ME_k/\partial\bar{\varepsilon} < 0$.

A marginal increase of $\bar{\varepsilon}$ affects ME_k according to:

$$\frac{\partial ME_k}{\partial \bar{\varepsilon}} = -\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\frac{1+(1-\tau)(A\alpha k^{\alpha-1} + \bar{\varepsilon} - \delta)}{1+(1-\tau)c} + [1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c} \tag{B.42}$$

An increase in $\bar{\varepsilon}$ increases default risk by $\varphi(\bar{\varepsilon})$. This discourages investment because it increases the probability that the *marginal* product of capital $A\alpha k^{\alpha-1}$ is lost. Because $\bar{\varepsilon}$ is chosen optimally, the first order condition (11) holds. Dividing (11) by k yields:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)\frac{\tau c}{1+(1-\tau)c} - \varphi(\bar{\varepsilon})(1+c)\frac{\tilde{b}-b}{k} = 0\,, \tag{B.43}$$

where $(\tilde{b}-b)/k$ is:

$$\frac{\tilde{b}-b}{k} = \frac{1+(1-\tau)\left(Ak^{\alpha-1}-\delta+\bar{\varepsilon}\right)}{1+(1-\tau)c} - \frac{b}{k} \tag{B.44}$$

The firm chooses $\bar{\varepsilon}$ taking into account part of the total expected costs of default which increase with leverage $\tilde{b}/k$ and therefore with the *average* product of capital $Ak^{\alpha-1}$. By combining (B.43) with (B.42), we derive:

$$\frac{\partial ME_k}{\partial \bar{\varepsilon}} = -\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\frac{1+(1-\tau)\left(A\alpha k^{\alpha-1}-\delta+\bar{\varepsilon}\right)}{1+(1-\tau)c} + \varphi(\bar{\varepsilon})\frac{1+c}{1+r}\frac{\tilde{b}-b}{k} \tag{B.45}$$

This expression is negative if and only if:

$$\frac{(1-\tau)Ak^{\alpha-1}\,(1-\alpha)}{1+(1-\tau)c} < \frac{b}{k} \tag{B.46}$$

The left hand side of B.46 is positive if $\alpha < 1$. If b is sufficiently small, this may imply that the inequality B.46 does not hold and $\partial ME_k/\partial\bar{\varepsilon} > 0$. For small positive values of α and b, a small decrease in $\bar{\varepsilon}$ may therefore discourage investment and mitigate rather than amplify changes in z. The reason is that the firm chooses $\bar{\varepsilon}$ by taking into account the expected loss of the *average* product of capital. At the same time, the firm chooses k by taking into account the expected loss of the *marginal* product of capital which is always lower than the *average* product. The marginal return net of taxes and default costs is maximized at a higher value of leverage and default risk than the average return.

This gap between the average and the marginal product disappears as $\alpha \to 1$. The left hand side of B.46 is approximately zero in this case and the inequality B.46 is always satisfied. In this case, the fact that $\bar{\varepsilon}$ falls in A (and z) amplifies the positive response of k with respect to A (and z). Through the same mechanism, the negative response of k to a fall in A is amplified by the resulting increase in $\bar{\varepsilon}$. The last step is to recall that optimal labor demand l is an increasing function of k, and output y is an increasing function of k and l. □

Proof of Proposition 4

Proof. The planner's objective is given by (12). With $\xi = 1$, the derivative of (12) with respect to k yields the same first order condition for capital as in (10). The stock of existing debt b does not appear in that expression. The planner's first order condition for $\bar{\varepsilon}$ is:

$$[1-\Phi(\bar{\varepsilon})]\frac{\partial \tilde{b}}{\partial \bar{\varepsilon}}\tau c-\varphi(\bar{\varepsilon})(1+c)\tilde{b}=0 \tag{B.47}$$

The planner's choice of $\bar{\varepsilon}$ differs from the firm's choice characterized by (11). In particular, it is independent of the stock of existing debt b. Because neither k depends on b, it follows that the planner's choice of $\tilde{b}$ (B.33) is independent of b.

Equation (B.47) is identical to the firm's first order condition (11) for the case without existing debt: $b = 0$. By applying the same reasoning as in the proof of Proposition 2, it is straightforward to show that leverage, default risk, and the credit spread are independent of z (for any value of b). □

C. Business Cycle Model

In Appendix C, we provide details of our solution method for the business cycle model with long-term and short-term debt (C.1), define key model variables and describe the construction of their empirical counterparts (C.2), provide results on parameter sensitivity (C.3), and lay out the setup of the frictionless model (*FL Model*, C.4) and the short-term debt model (*STD Model*, C.5) used in Figure 7, as well as the constrained efficient allocation used in Figure 8 (*Constr. Eff.*, C.6).

C.1. Solution Method

This appendix presents a detailed description of the computational procedure that is used to find the equilibrium of the benchmark model with long-term and short-term debt laid out in Section 4.6.

We compute the equilibrium of a dynamic open economy business cycle model with a given international risk-free rate r and an endogenous wage w. Due to distortionary taxes, default, and lack of commitment, the equilibrium allocation is inefficient. One cannot directly compute a centralized solution but must solve the decentralized equilibrium allocation. All agents take the factor prices r and w as given. The aggregate state of the economy S consists of aggregate productivity z' and the aggregate stock of existing debt $B : S = (z', B)$. Given the current aggregate state S and a law of motion $S' = F(S)$, agents forecast current and future values of the wage $w(S)$. There is a constant unit mass of ex-ante identical firms. The endogenous state variable of an individual firm is b. In equilibrium, we therefore have: $B' = b' = (1-\gamma)\tilde{b}^L$.

We find the global solution to the dynamic firm problem in (24) and the equilibrium defined in Section 4.6 by value function iteration with interpolation. The key difficulty consists in finding the equilibrium price of long-term debt p^L. Optimal firm behavior depends on p^L which itself depends on the expected future price of long-term debt which in turn depends on future firm behavior. We solve this fixed point problem by computing the equilibrium of a finite-horizon economy. Starting from a final date T, we iterate backward until all prices and quantities have converged. We then treat the first-period equilibrium allocation as the equilibrium of the infinite-horizon economy.

Given that the continuation value of an individual firm $V((1-\gamma)\tilde{b}^L, S')$ and the future price of long-term debt $g((1-\gamma)\tilde{b}^L, S')$ are zero in the final period T, this is a suitable starting point for the iteration process.

The computational procedure is implemented in Matlab. To compute the solution of the firm problem (24), we create grids for the endogenous state of an individual firm b, the endogenous state of the aggregate economy B, and the exogenous state z'. For b, we use a linear grid with $\#_b$ grid points and support $[0, \bar{b}]$, where $\bar{b}$ is set sufficiently high such that $(1-\gamma)\tilde{b}^L(b,S) < \bar{b}$ for all firm states (b,S). The grid for B is identical. The stochastic process of $\ln z'$ is approximated using a grid with $\#_z$ grid points and a transition matrix Π constructed following the Rouwenhorst method as in Kopecky and Suen (2010). The results presented in the paper are computed using $\#_b = 10$, $\bar{b} = 0.3$, and $\#_z = 5$. This yields a state space (b,S) with $10 \times 10 \times 5 = 500$ grid points. Convergence is typically achieved after about 300 periods. Thanks to interpolation, the computational procedure is robust to variations in $\#_b$ and $\#_z$. For instance, we have carried out computations using $\#_b = 8$, $\#_b = 20$, or $\#_z = 3$. In all cases, the results are highly similar.

The algorithm proceeds as follows:

1. Start at final date T. Set the value function $V_T(b_{iT}, S_T) = 0$ and the price function of long-term debt $g_T(b_{iT}, S_T) = 0$ for all (b_{iT}, S_T).

2. At the end of period $T-1$, apply the following steps:

   a) Given the aggregate state $S_{T-1} = (z_T, B_{T-1})$, guess aggregate firm capital $K_T(S_{T-1})$.

   b) Given $K_T(S_{T-1})$, aggregate labor demand is found by using firms' first order condition for labor (19): $L_T^d = (\zeta(1-\psi)z_T K_T^{\psi\zeta}/w_T)^{1/(1-\zeta(1-\psi))}$. Aggregate labor supply follows from (26): $L_T^s = w_T^{1/\theta}$. Using labor market clearing, compute the equilibrium wage w_T.

   c) The guess for $K_T(S_{T-1})$ has provided us with an initial guess for $w_T(S_{T-1})$. Given this guess for w_T, solve the firm-level problem (24) for the firm state (b_{iT-1}, S_{T-1}) with $b_{iT-1} = B_{T-1}$.

      - In this final period, no new long-term debt is issued and all existing long-term debt matures at time T: $\tilde{b}_{iT}^L = b_{iT-1}$ with $\gamma = 1$. The firm problem at the end of period $T-1$ can be re-written in terms of only two choice variables: k_{iT} and $\tilde{b}_{iT}^S$. Given z_T, k_{iT}, and w_T, individual labor demand l_{iT} is given by (19). Using z_T, k_{iT}, l_{iT}, $\tilde{b}_{iT}^S$, $\tilde{b}_{iT}^L$, and w_T, we can compute firm output y_{iT}, the asset value in case of default $\underline{q}(\varepsilon_{iT})$, and the threshold value $\bar{\varepsilon}_{iT}$. This determines the default probability $\Phi(\bar{\varepsilon}_{iT}) = \frac{1}{2}\left[1 + \text{erf}\left(\bar{\varepsilon}_{iT}/(\sigma_\varepsilon\sqrt{2})\right)\right]$.

      - Using $\bar{\varepsilon}_{iT}$, $\Phi(\bar{\varepsilon}_{iT})$, $\tilde{b}_{iT}^S$, $\tilde{b}_{iT}^L$, and $\underline{q}_{iT}$, the price of short-term debt p_{iT-1}^S is given by (22). The fact that ε_{iT} is drawn from a continuous probability distribution implies that the threshold value $\bar{\varepsilon}_{iT}$ and the bond price p_{iT-1}^S are continuous as well.

- Using these constraints, numerically solve for the combination of firm capital k_{iT} and short-term debt $\tilde{b}^S_{iT}$ that maximizes the firm objective in (24). None of the firm choices is restricted to lie on a grid. The dividend payout constraint $\underline{e}$ is set such that it is not binding in equilibrium. The exact value of $\underline{e}$ does not affect equilibrium variables.

  Note that the bond price p^S_{it} is pinned down by the firm's current policy. The equilibrium bond price and firm policy are computed in a single step. It is not necessary to compute bond prices for all possible firm actions in an 'outer loop' before computing optimal firm policy in a subsequent 'inner loop'. Avoiding this 'inner loop-outer loop' procedure reduces the number of necessary computations.

d) Compare the solution of the firm problem for capital k_{iT} to the guess K_T. Because there is a constant unit mass of ex-ante identical firms, these two must be identical in equilibrium. In this case, aggregate labor supply L^s_T is equal to aggregate labor demand $L^d_T = l_{iT}$ at the wage w_T. The labor market clears. If the absolute distance between k_{iT} and K_T is below a pre-defined tolerance level, continue to the next step, otherwise update K_T and return to step 2b.

e) Once we have found the equilibrium wage $w(S_{T-1})$ and the solution to the firm problem (24) for the firm state (b_{iT-1}, S_{T-1}) with $b_{iT-1} = B_{T-1}$, we compute the solution to (24) for all firm states (b_{iT-1}, S_{T-1}) with $b_{iT-1} \neq B_{T-1}$. The equilibrium wage $w(S_{T-1})$ is held constant during this step because it only depends on the aggregate state S_{T-1}.

f) Use these results to store the value function $V_{T-1}(b_{iT-1}, S_{T-1})$ and the price function of long-term debt $g_{T-1}(b_{iT-1}, S_{T-1})$ in all firm states (b_{iT-1}, S_{T-1}).

3. In all periods $t < T-1$, apply the following steps. They closely follow the procedure from period $T-1$, with the addition of long-term debt $\tilde{b}^L_{it+1}$ as a new choice variable and the law of motion $S_{t+1} = F_t(S_t)$.

   a) Given the aggregate state $S_t = (z_{t+1}, B_t)$, guess aggregate firm capital $K_{t+1}(S_t)$.

   b) Given $K_{t+1}(S_t)$, compute the equilibrium wage w_{t+1} as in step 2b above.

   c) Guess a value for the future aggregate state B_{t+1}. Together with the stochastic process of z_{t+1}, this yields a candidate law of motion for the aggregate state $S_{t+1} = F_t(S_t)$.

      i. Given the current guess for w_{t+1} and the candidate law of motion $S_{t+1} = F_t(S_t)$, solve the firm-level problem (24) for the firm state (b_{it}, S_t) with $b_{it} = B_t$.

      - The firm problem at the end of period t can be re-written in terms of three choice variables: capital k_{it+1}, short-term debt $\tilde{b}^S_{it+1}$, and long-term debt $\tilde{b}^L_{it+1}$. Compute individual labor demand l_{it+1}, firm output y_{it+1}, and the asset value in case of default $\underline{q}(\varepsilon_{it+1})$. The solution to the equilibrium of period $t+1$ (as computed previously) provides the value function $V_{t+1}((1-\gamma)\tilde{b}^L_{it+1}, S_{t+1})$. Use it together with $S_{t+1} = F_t(S_t)$ to compute the threshold value $\bar{\varepsilon}_{it+1}$ and the default probability $\Phi(\bar{\varepsilon}_{it+1})$. As above, none of the firm choices is restricted to lie on a grid. To compute the exact solution of $\bar{\varepsilon}_{it+1}$, off-grid values of $V_{t+1}((1-\gamma)\tilde{b}^L_{it+1}, S_{t+1})$ are approximated by cubic interpolation.

- The price of short-term debt p^S_{it} is computed as above. The key equilibrium variable of the model is the price of long-term debt p^L_{it} as given by (23). It not only depends on the firm's current behavior but also on the future price of long-term debt which in turn depends on future firm behavior. The solution to the equilibrium of period $t+1$ (as computed previously) provides the future long-term debt price $g_{t+1}((1-\gamma)\tilde{b}^L_{it+1}, S_{t+1})$. Use it together with $S_{t+1} = F_t(S_t)$ to compute p^L_{it}. To compute the exact solution of p^L_{it}, off-grid values of $g_{t+1}((1-\gamma)\tilde{b}^L_{it+1}, S_{t+1})$ are approximated by cubic interpolation. The fact that ε_{it+1} is drawn from a continuous probability distribution implies that p^S_{it} and p^L_{it} are continuous as well.
- Using these constraints, numerically solve for the combination of k_{it+1}, $\tilde{b}^S_{it+1}$, and $\tilde{b}^L_{it+1}$, that maximizes the firm objective in (24). As above, none of the firm choices is restricted to lie on a grid. The dividend payout constraint $\underline{e}$ is set such that it is not binding in equilibrium. The exact value of $\underline{e}$ does not affect equilibrium variables.

  Note that the equilibrium bond prices p^S_{it} and p^L_{it} are pinned down by the firm's current and future policy. Equilibrium bond prices and firm policy are computed in a single step. It is not necessary to compute bond prices for all possible firm actions in an 'outer loop' before computing optimal firm policy in a subsequent 'inner loop'. Avoiding this 'inner loop-outer loop' procedure reduces the number of necessary computations.

ii. Compare the solution of the firm problem for the future stock of existing debt $b_{it+1} = (1-\gamma)\tilde{b}^L_{it+1}$ to the guess B_{t+1}. Because there is a constant unit mass of ex-ante identical firms, these two must be identical in equilibrium. If the absolute distance between b_{it+1} and B_{t+1} is below a pre-defined level of tolerance, continue to step 3d, otherwise update the guess B_{t+1} and the candidate law of motion $S_{t+1} = F_t(S_t)$, and return to step 3(c)i.

d) Compare the solution of the firm problem for capital k_{it+1} to the guess K_{t+1}. Because there is a constant unit mass of ex-ante identical firms, these two must be identical in equilibrium. In this case, the labor market clears. If the absolute distance between k_{it+1} and K_{t+1} is below a pre-defined tolerance level, continue to the next step, otherwise update K_T and return to step 3c.

e) Once we have found the equilibrium wage $w(S_t)$ and the solution to the firm problem (24) for the firm state (b_{it}, S_t) with $b_{it} = B_t$, we compute the solution to (24) for all firm states (b_{it}, S_t) with $b_{it} \neq B_t$.

f) Use these results to store the value function $V_t(b_{it}, S_t)$ and the price function of long-term debt $g_t(b_{it}, S_t)$ in all firm states (b_{it}, S_t).

g) If the absolute distances between $V_t(b_{it}, S_t)$ and $V_{t+1}(b_{it+1}, S_{t+1})$, and between $g_t(b_{it}, S_t)$ and $g_{t+1}(b_{it+1}, S_{t+1})$ are above a pre-defined level of tolerance, continue with period $t-1$. If the absolute distances are below the tolerance level, the equilibrium allocation is found.

C.2. Model Moments and Empirical Moments

In this section, we define key model variables (C.2.1) and describe the construction of their empirical counterparts (C.2.2 and C.2.3).

C.2.1. Model Moments

Table 4 defines key model variables. Detailed derivations are provided below.

The total amount of firm debt is the present value of future debt payments discounted at the riskless rate r:

$$\begin{aligned} D &= \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L + (1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ... \\ &= \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L \sum_{j=0}^{\infty}\left(\frac{1-\gamma}{1+r}\right)^j = \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\tilde{b}^L \end{aligned} \tag{C.48}$$

The long-term debt share of a given firm is the present value of debt payments due more than one year from today divided by the total amount of firm debt D:

$$\begin{aligned} &\frac{1}{D}\left((1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ...\right) \\ =&\frac{1}{D}\frac{\gamma+c}{\gamma+r}\frac{1-\gamma}{1+r}\tilde{b}^L \end{aligned} \tag{C.49}$$

The Macaulay duration is the weighted average term to maturity of the cash flows from a bond divided by the price:

$$\mu = \frac{1}{p_r^L}\sum_{j=1}^{\infty} j(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{p_r^L}\frac{1+r}{(\gamma+r)^2} \tag{C.50}$$

where p_r^L is the price of a riskless long-term bond:

$$p_r^L = \sum_{j=1}^{\infty}(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{r+\gamma} \tag{C.51}$$

Table 4: Business Cycle Model - Variables

GDP	$y_t - f - \xi \int_{-\infty}^{\bar{\varepsilon}_t} \underline{q}_t \, \varphi(\varepsilon)\, d\varepsilon - H(\tilde{b}^S_{t+1}, \tilde{b}^L_{t+1}, b_{t+1})$
Total debt	$D \equiv \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\tilde{b}^L$
Leverage: Firm debt / Firm assets	D/k
Long-term debt share	$\frac{1}{D}\frac{\gamma+c}{\gamma+r}\frac{1-\gamma}{1+r}\tilde{b}^L$
Macaulay duration	$\frac{1+r}{\gamma+r}$
Default rate	$\Phi(\bar{\varepsilon})$
Short-term spread	$sp^S \equiv \frac{1+c}{p^S} - (1+r)$
Long-term spread	$sp^L \equiv \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)$
Issuance weighted average credit spread	$\frac{\tilde{b}^S}{\tilde{b}^S+\tilde{b}^L-b} \times sp^S + \frac{\tilde{b}^L-b}{\tilde{b}^S+\tilde{b}^L-b} \times sp^L$
Term structure	$sp^L - sp^S$

It follows for the Macaulay duration:

$$\mu = \frac{1+r}{\gamma+r} \tag{C.52}$$

The short-term spread compares the gross return (in the absence of default) from buying a short-term bond with the riskless rate:

$$\frac{1+c}{p^S} - (1+r) \tag{C.53}$$

The long-term spread compares the gross return (in the absence of default and assuming p^L is constant) from buying a long-term bond with the riskless rate:

$$\frac{\gamma + c + (1-\gamma)p^L}{p^L} - (1+r) = \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r) \tag{C.54}$$

C.2.2. Empirical Moments Table 2

Leverage and the long-term debt share are from Compustat for the years 1984-2015. We exclude financial firms (SIC codes 6000-6999) and utilities (SIC 4900-4949) as well as firm-year observations with an ISO Currency Code different from US Dollar.

Leverage is the average of the aggregate book value of total debt (annual data item 34 + data item 9) over the aggregate book value of total firm assets (at historical cost, data item 6). The long-term debt share is the average of aggregate debt with remaining term to maturity of more than one year (data item 9) over aggregate total firm debt (data item 34 + data item 9).

The average credit spread is from Adrian et al. (2013), Table 2, who use micro data on new debt issuances of various maturities by US corporations 1998-2010. We target the issuance amount weighted average spread on all loan and bond issuances. The model counterpart is the issuance weighted average of the credit spread on short-term debt and long-term debt as defined in Table 4.

C.2.3. Empirical Moments Table 3

Annual data on GDP and total debt in Table 3 is from the Flow of Funds 1984-2015. This is the same data as used in Figure 1 and Figure 2 (see Appendix A.2 for details). The moments for total debt are calculated using data for all non-financial firms. Results are highly similar if we restrict ourselves to the corporate sector.

Leverage, b/k, and the long-term debt share are calculated using annual Compustat data 1984-2015. To facilitate comparison with the Flow of Funds data, we only include Compustat firm-year observations which are reported in December of a given year. We exclude financial firms (SIC codes 6000-6999) and utilities (SIC 4900-4949) as well as firm-year observations with an ISO Currency Code different from US Dollar. Furthermore, we exclude observations which report negative *Firm Debt* (data item 34 + data item 9) or *Sales* (data item 12), as well as those which do not report information on *Long-term Debt* (data item 9), *Firm Debt*, or *Sales*.

Leverage is the average of the aggregate book value of total *Firm Debt* over the aggregate book value of total firm assets (at historical cost, data item 6). b/k at the end of year t is the CPI-deflated stock of aggregate debt at the end of year $t-1$ with remaining term to maturity of more than one year (data item 9) divided by CPI-deflated total assets at the end of year t (data item 6). The long-term debt share is the average of aggregate debt at the end of year t with remaining term to maturity of more than one year (data item 9) over aggregate total *Firm Debt* at the end of year t (data item 34 + item 9).

The default rate is from from Giesecke et al. (2014). It denotes the total defaulted value of US corporate debt over total par value at annual frequency (1984-2012).

Data on credit spreads is from the FRED database of the St. Louis Fed 1997-2015. We use this data source because it provides time series on credit spreads broken down by different maturities. *Credit Spread* is the ICE BofAML US Corporate Master Option-Adjusted Spread. This is a market capitalization-weighted average of option-adjusted spreads of US investment grade corporate bonds (remaining maturity above one year, minimum amount outstanding of 250 million USD, currently not in default) relative to a spot Treasury curve. The model counterpart is the issuance weighted average of the credit spread on short-term debt and long-term debt.

Term Structure is the difference between the ICE BofAML US Corporate 7-10 Year Option-Adjusted Spread (remaining term to maturity between seven and ten years) and the ICE BofAML US Corporate 1-3 Year Option-Adjusted Spread (remaining term to maturity less than three years). A maturity between seven and ten years roughly matches the average maturity of a long-term bond in our model with $\gamma = 0.1284$. The maturity of less than three years is the shortest maturity available in FRED.

Table 5: Parameter Sensitivity - Model Moments

	Benchmark	$\gamma = 0.4$	$\sigma_\varepsilon = 0.75$	$\xi = 0.45$	$\eta = 0$
Leverage: Debt / Assets	29.3%	21.6%	25.0%	33.1%	21.5%
Long-term debt share	75.4%	58.2%	79.2%	66.0%	26.8%
Average credit spread	2.3%	1.8%	3.4%	2.5%	1.7%
Default rate	2.6%	1.9%	3.4%	3.3%	1.8%
GDP volatility	3.0%	2.8%	3.1%	3.2%	2.7%

Note: Each column corresponds to a distinct set of parameter values. *Benchmark* is the equilibrium of the benchmark model with long-term and short-term debt (Section 4.6) using the parameter values given in Table 1. All other columns use the same set of parameter values with the exception of the indicated model parameter. *Average credit spread* is the issuance weighted average of the credit spread on short-term debt and long-term debt as defined in Table 4. *GDP volatility* is the standard deviation of linearly detrended annual ln GDP.

C.3. Parameter Sensitivity

In this section, we provide results on the sensitivity of key model moments with respect to parameter values. Table 5 and Figure 12 present results from the benchmark model with long-term and short-term debt (Section 4.6) for different sets of parameter values. *Benchmark* corresponds to the parameter values calibrated to US data given in Table 1. In addition, results are shown for four different sets of parameter values. In each case, only the indicated parameter value differs from the values given in Table 1.

- $\gamma = 0.4$: The main benefit of borrowing at long maturities is that fewer bonds need to be issued each period to maintain a given amount of leverage. Using long-term debt therefore saves issuance costs. Increasing the repayment rate of long-term debt from 0.1284 (average Macaulay duration 6.5 years) to 0.4 (average duration 2.4 years) implies that firms need to roll-over their long-term debt at higher frequency which reduces the benefit of borrowing at long maturities. The equilibrium share of long-term debt falls. The lower stock of outstanding long-term debt induces firms to reduce average leverage relative to the *Benchmark* case. The average default rate and credit spreads fall. As shown in Figure 12, total firm debt co-moves more strongly with contemporaneous output at the higher value of γ. This reduces the volatility of leverage and credit spreads and thereby lowers GDP volatility from 3.0% to 2.8%.[24]

- $\sigma_\varepsilon = 0.75$: An increase in the standard deviation of the firm-specific capital quality shock ε from 0.652 to 0.75 leads to higher default risk. Firms respond by reducing average leverage. However, higher default risk increases the sensitivity of the long-term bond price p^L with respect to firm behavior. Firms' incentive to increase

[24]For comparison, GDP volatility is 2.5% in the constrained efficient allocation, the frictionless model, and the short-term debt model.

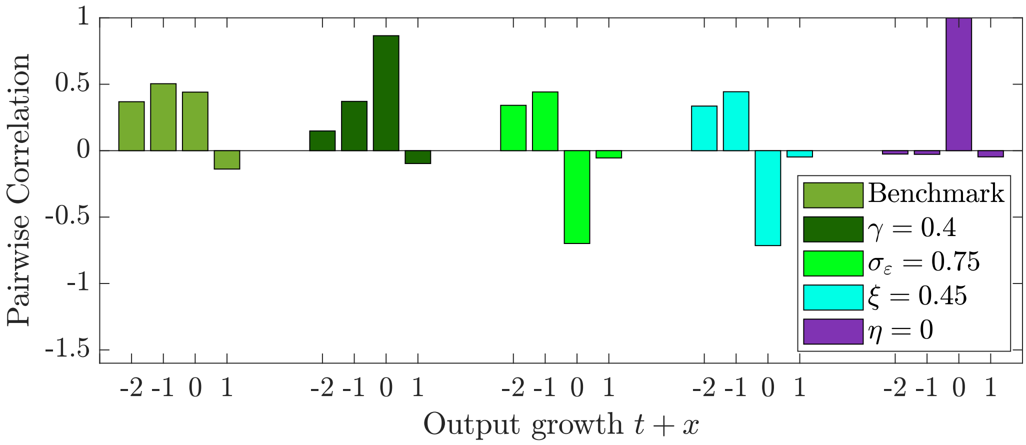

Figure 12: Parameter Sensitivity - Correlations Firm Credit Growth t and Output Growth $t + x$

Note: Bars show pairwise correlations between annual growth of total firm debt at the end of year t and GDP growth in year $t + x$. Each group of bars corresponds to a distinct set of parameter values. *Benchmark* is the equilibrium of the benchmark model with long-term and short-term debt (Section 4.6) using the parameter values given in Table 1. All other four-bar groups use the same set of parameter values with the exception of the indicated model parameter.

leverage and default risk during a downturn at the expense of existing creditors is reinforced. As shown in Figure 12, the increase in leverage during a downturn can be strong enough for debt to *rise* when output falls. The contemporaneous correlation between debt and output becomes negative. At the same time, the resulting strong increase in credit spreads during the downturn amplifies the fall in output and leads to higher GDP volatility.

- $\xi = 0.45$: A reduction in default costs from $\xi = 0.669$ to $\xi = 0.45$ shifts the trade-off between the tax advantage of debt and expected default costs in favor of higher average leverage and default risk. As explained above, higher default risk increases the sensitivity of the long-term bond price p^L with respect to firm behavior. This amplifies the counter-cyclical behavior of leverage and credit spreads and translates into higher GDP volatility.

- $\eta = 0$: A reduction in the debt issuance cost from 0.0077 down to zero implies that debt roll-over is now costless. This reduces the disadvantage of borrowing at short maturities. The equilibrium share of long-term debt falls. The lower stock of outstanding long-term debt leads to reduced average leverage, default risk, and credit spreads. As shown in Figure 12, the lag in total debt with respect to output disappears. Without 'slow debt', GDP volatility falls to 2.7%.

  As shown in Table 5, even in the absence of roll-over costs firms issue small positive amounts of long-term debt in this model. In Jungherr and Schott (2020) we show that, ceteris paribus, firms prefer owing a given stock of debt in the form of long-term rather than short-term bonds. The reason is that the positive probability of future default lowers the expected repayment of long-term debt from the firm to existing creditors. Because of default risk, firms discount the future at a higher rate than creditors.

C.4. Business Cycle Model - Frictionless

Figure 7 displays impulse response functions of a frictionless open economy business cycle model without default costs, taxes, or debt issuance costs. The Modigliani-Miller irrelevance result holds in this environment.

C.4.1. Setup

There is a unit mass of ex-ante identical firms. The production technology is the same as in the benchmark model with long-term debt. Firm earnings are given as

$$z_t \left(k_{it}^{\psi} l_{it}^{1-\psi}\right)^{\zeta} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f \tag{C.55}$$

The firm-specific idiosyncratic earnings shock ε_{it} is i.i.d. and follows a probability distribution $\varphi(\varepsilon)$ with zero mean. As in the long-term debt model, productivity evolves according to: $\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t$, where ϵ_t is white noise with standard deviation σ_z. Capital k_{it} and labor l_{it} are chosen at the end of period $t-1$ after z_t is realized.

Just as before, there is a representative household with GHH preferences over consumption C_t and labor L_t:

$$u\left(C_t - \frac{L_t^{1+\theta}}{1+\theta}\right), \tag{C.56}$$

with $u(\cdot)$ being strictly increasing and concave, and $\theta > 0$.

C.4.2. Optimal Firm Behavior

Conditional on k_{it} and the realization of z_t, an individual firm chooses labor to maximize static profits:

$$l_{it} = \left(\frac{(1-\psi)\zeta z_t k_{it}^{\psi\zeta}}{w_t}\right)^{\frac{1}{1-(1-\psi)\cdot\zeta}} \tag{C.57}$$

Optimal capital demand solves:

$$\max_{k_{it}} - k_{it} + \frac{1}{1+r}\int_{-\infty}^{\infty}\left[(1-\delta)k_{it} + z_t\left(k_{it}^{\psi} l_{it}^{1-\psi}\right)^{\zeta} + \varepsilon k_{it} - w l_{it} - f\right]\varphi(\varepsilon)d\varepsilon \tag{C.58}$$

subject to (C.57). We define the profitability term A_t:

$$A_t = z_t^{\frac{1}{1-(1-\psi)\cdot\zeta}} \cdot \left(\frac{(1-\psi)\cdot\zeta}{w_t}\right)^{\frac{(1-\psi)\cdot\zeta}{1-(1-\psi)\cdot\zeta}} - w_t\left(\frac{(1-\psi)\cdot\zeta}{w_t}\right)^{\frac{1}{1-(1-\psi)\cdot\zeta}} \tag{C.59}$$

This implies for optimal capital demand:

$$k_{it} = \left(\frac{A_t \psi \zeta}{(r+\delta)[1-(1-\psi)\zeta]} \right)^{\frac{1-(1-\psi)\zeta}{1-\zeta}} \tag{C.60}$$

C.4.3. Equilibrium

Definition: Competitive Equilibrium. Given a realization of z_t, a competitive equilibrium consists of *(i)* quantities of capital k_{it} and labor l_{it}, and *(ii)* a wage rate w_t, such that:

1. k_{it} and labor l_{it} satisfy (C.60) and (C.57)
2. The labor market clears:
$$w_t^{\frac{1}{\theta}} = l_{it}$$

The parameters r, ζ, ψ, δ, θ, σ_z, and ρ_z are left unchanged with respect to the benchmark model.

C.5. Business Cycle Model - Short-term Debt

Figure 6, Figure 7, and Table 3 report results for a business cycle model of production, firm financing, and costly default in which firms use only short-term debt. This short-term debt model shares most of the setup with the long-term debt model described in Section 4. A key difference is that firms cannot issue long-term debt now.

C.5.1. Optimal Firm Behavior

Given a realization of aggregate productivity z', an individual firm chooses a policy vector $\{k, l, \tilde{e}, \tilde{b}^S, \bar{\varepsilon}\}$ which solves

$$\max_{\{k,l,\tilde{e},\tilde{b}^S,\bar{\varepsilon}\}} -\tilde{e} + \frac{1}{1+r} \int_{\bar{\varepsilon}}^{\infty} \left[k - \tilde{b}^S + (1-\tau) \left[y + \varepsilon k - w(z')l - \delta k - f - c\tilde{b}^S \right] \right] \varphi(\varepsilon) d\varepsilon \tag{C.61}$$

s.t.: $y = z' \left(k^{\psi} l^{1-\psi} \right)^{\zeta}$

$$l = \left(\frac{\zeta(1-\psi) z' k^{\psi\zeta}}{w(z')} \right)^{\frac{1}{1-\zeta(1-\psi)}}$$

$$\bar{\varepsilon}: \quad k - \tilde{b}^S + (1-\tau) \left[y + \bar{\varepsilon} k - w(z')l - \delta k - f - c\tilde{b}^S \right] = 0$$

$$k = \tilde{e} + p^S \tilde{b}^S$$

$$p^S = \frac{1}{1+r} \left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^S} \int_{-\infty}^{\bar{\varepsilon}} \underline{q} \; \varphi(\varepsilon) d\varepsilon \right]$$

Table 6: Short-term Debt Model - Parametrization

Parameter	Description	Value
f	fixed cost	0.16526
σ_ε	st. dev. idiosyncratic shock	0.67
ξ	default cost	0.194

C.5.2. Equilibrium

Definition: Competitive Equilibrium. Given a realization of z', a competitive equilibrium consists of *(i)* a firm policy $\{k, l, \tilde{e}, \tilde{b}^S, \bar{\varepsilon}\}$, and *(ii)* a wage rate $w(z')$, such that:

1. $\{k, l, \tilde{e}, \tilde{b}^S, \bar{\varepsilon}\}$ solve the firm problem C.61

2. The labor market clears:
$$w(z')^{\frac{1}{\theta}} = l$$

Most parameters are left unchanged with respect to the benchmark model. We adjust the values of σ_ε and ξ in order to match the same average leverage ratio (29.3%) and the same average credit spread (2.3%) as in the benchmark model. We also change the value of the fixed cost of operation f in order to maintain zero firm profits on average. Table 6 summarizes all parameter changes with respect to Table 1.

C.6. Business Cycle Model - Constrained Efficiency

The only difference between the recursive competitive equilibrium described in Section 4.6 and the equilibrium for the constrained efficient case lies in the nature of the firm problem. The value function $V(b, S)$ in (24) is replaced by the value $W(b, S)$ which solves:

$$W(b,S) = \max_{\phi(b,S)=\left\{k,\tilde{e},\tilde{b}^S,\tilde{b}^L,\bar{\varepsilon}\right\}} \quad p^L b - T(b,S) - \tilde{e} + \frac{1}{1+r}\mathbb{E}_{S'|S}\left\{\int_{\bar{\varepsilon}}^{\infty}\left[q' + W\left((1-\gamma)\tilde{b}^L, S'\right)\right]\varphi(\varepsilon)d\varepsilon\right\} \quad \text{(C.62)}$$

$$\text{s.t.:} \quad q' = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\left[y + \varepsilon k - w(S)l - \delta k - f - c(\tilde{b}^S + \tilde{b}^L)\right]$$

$$y = z'\left(k^{\psi}l^{1-\psi}\right)^{\zeta}$$

$$l = \left(\frac{\zeta(1-\psi)z'k^{\psi\zeta}}{w(S)}\right)^{\frac{1}{1-\zeta(1-\psi)}}$$

$$\bar{\varepsilon}: \quad q' + W\left((1-\gamma)\tilde{b}^L, S'\right) = 0$$

$$k = \tilde{e} + p^S\tilde{b}^S + p^L(\tilde{b}^L - b) - H(\tilde{b}^S, \tilde{b}^L, b)$$

$$p^S = \frac{1}{1+r}\mathbb{E}_{S'|S}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\ \varphi(\varepsilon)d\varepsilon\right]$$

$$p^L = g(b,S) = \frac{1}{1+r}\mathbb{E}_{z'|z}\left[[1-\Phi(\bar{\varepsilon})]\left[\gamma + c + (1-\gamma)\,g\left((1-\gamma)\tilde{b}^L, S'\right)\right] + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\ \varphi(\varepsilon)d\varepsilon\right]$$

The state-contingent tax $T(b,S)$ in (C.62) is specified such that in equilibrium: $T(b,S) = p^L b$. This makes sure that $W(b,S)$ differs from the value $V(b,S)$ in the decentralized model only because of different firm behavior.

D. Empirical Literature on Seniority and Covenants

Market participants try to mitigate the commitment problem generated by risky long-term debt through various contracting features such as seniority structures or debt covenants. While a formal analysis of these instruments is beyond the scope of this paper, in the following we provide a brief overview of the empirical literature on this topic.

Seniority: The majority of U.S. corporate bonds consists of senior unsecured bonds (68%). Subordinated debt makes up for only 5% in value (Gomes et al., 2016), and less than 25% of the number of bond issues (Billett, King, and Mauer, 2007). Secured debt is an alternative way to grant priority to certain debt claims. Secured debt is less than 20% of the number of bond issues (Billett et al., 2007), and less than 20% of the value of issuance (Benmelech, Kumar, and Rajan, 2020). In the cross-section of firms, the share of secured debt is higher for firms with higher default risk (Benmelech et al., 2020).

Covenants: Firms exert a negative externality on existing creditors if they increase default risk by issuing additional debt and by reducing equity injections or increasing dividend payout. The empirical literature finds that less than 25% of U.S. investment grade corporate bonds include covenants which restrict the issuance of additional debt, and less than 20% feature restrictions of firms' dividend policy. Nash, Netter, and Poulsen (2003) document that 15.66% of 364 investment grade bond issues in 1989 and in 1996 feature restrictions on additional debt. 8.24% include restrictions of the firm's dividend policy.[25] In a sample of 100 bond issues between 1999-2000, Begley and Freedman (2004), Table 2, p. 24, report that 9% contain additional borrowing restrictions. The percentage for dividend restrictions is identical (9%). Billett et al. (2007), Table III, p. 707, calculate that 22.8% of 15,504 investment grade bond issued between 1960 and 2003 had a covenant which restricts future borrowing of identical (or lower) seniority. 17.1% had a covenant which restricts dividend policy.[26] Reisel (2014), Table 4, p. 259, finds in a sample of 4,267 bond issues from 1989 - 2006 that 5.9% of investment grade bonds feature covenants which restrict additional borrowing or the firm's dividend policy. In the cross-section of firms, these covenants are more common for junk bonds than for investment grade bonds (Billett et al., 2007; Green, 2018).

While debt covenants are relatively infrequent for investment grade corporate bonds, they are widely used in bank lending. Roberts and Sufi (2009) document that covenant violations are frequent and that they impact firm behavior. However, the mere existence of a restrictive covenant may not be sufficient to grant protection to lenders. Covenants are frequently weakened by "fine print" clauses (Ivashina and Vallee, 2019), and in about two thirds of all covenant violations creditors take no action and there are no consequences for the borrowing firm (Roberts and Sufi, 2009).

[25] Of the 496 bonds considered in their Compustat sample, 120 feature additional debt restrictions (Table 3, p. 218). Of those, 57 bonds are investment grade (Table 4, p.220). It follows that out of a total of 364 investment grade bonds (Table 2, p.216), 15.66% feature additional debt restrictions. Out of the full sample, 99 bonds include restrictions of the firm's dividend policy (Table 3, p. 218). Of those, 30 bonds are investment grade (Table 4, p.220). It follows that 8.24% of the investment grade bonds in the sample feature dividend restrictions.

[26] Future borrowing of identical (or lower) seniority is restricted by funded debt restrictions (4.5%), subordinate debt restrictions (0.8%), and total leverage tests (17.5%). Issuance of secured debt (with effective priority over existing debt) is more frequently restricted than unsecured debt. Dividend policy is restricted by dividend payment restrictions (12.1%) and share repurchase restrictions (5.0%).